Stocks & Flows 2: First Order Delays & Aging Chains

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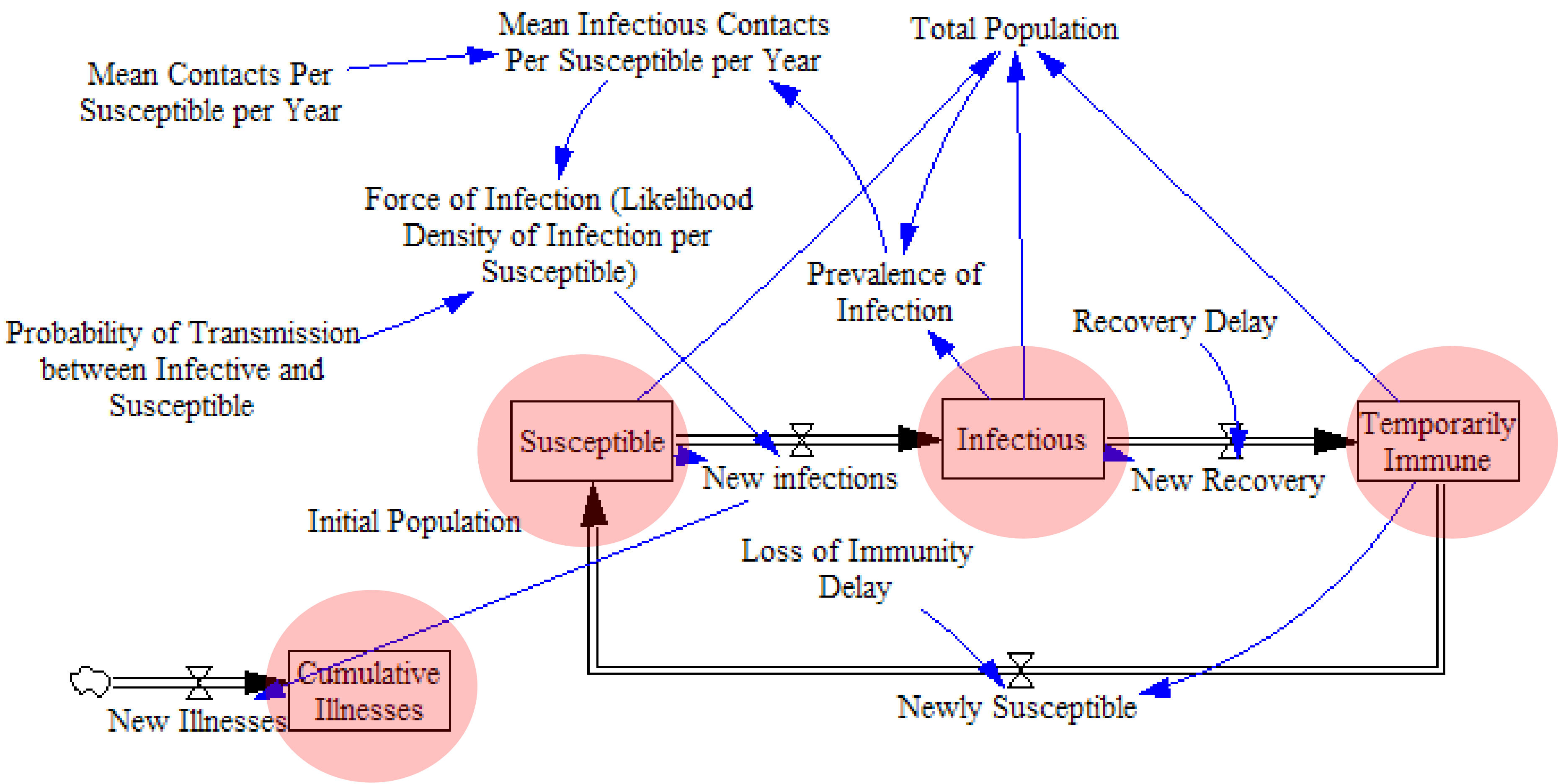
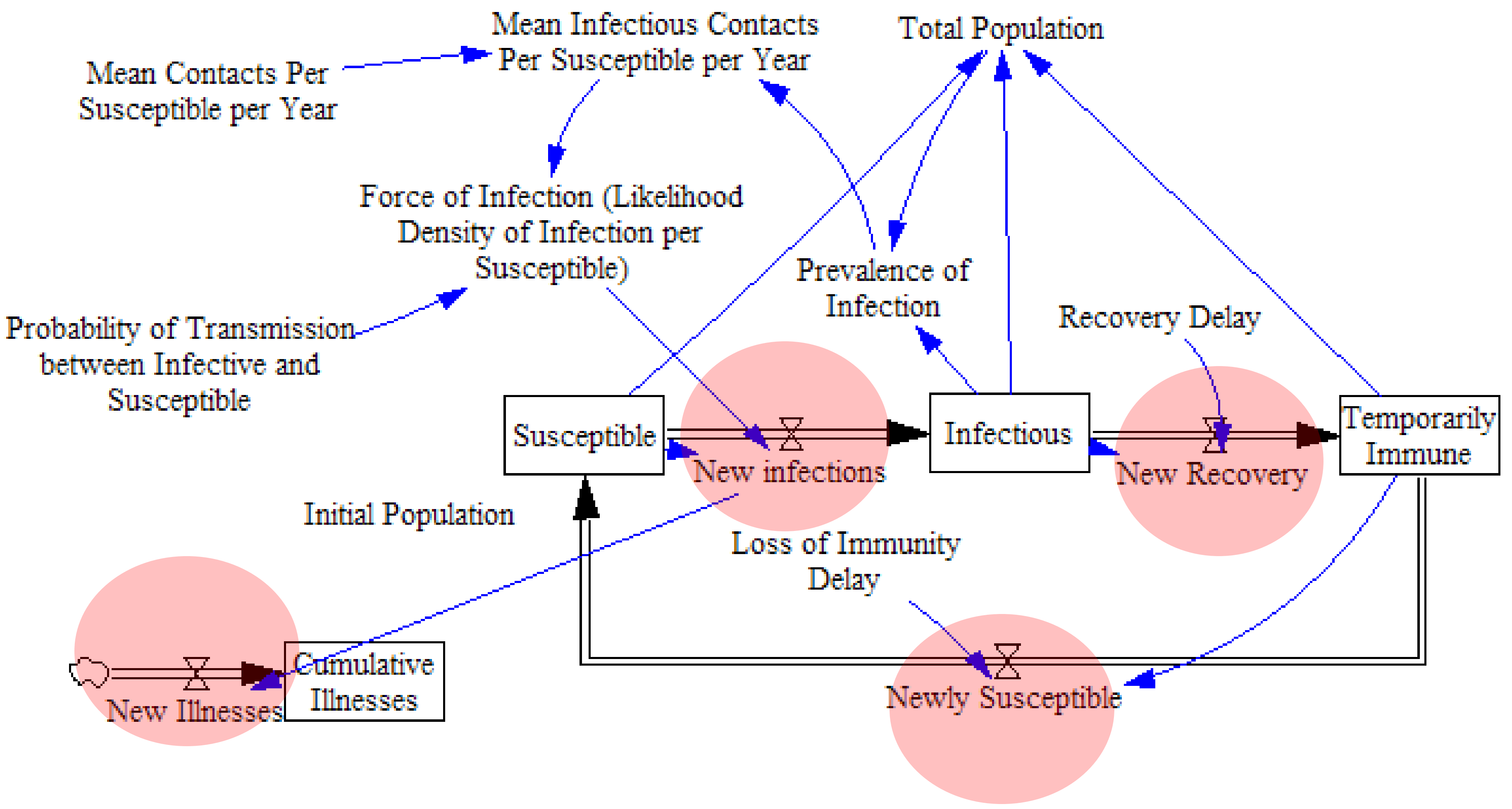
CMPT 858

February 1, 2011

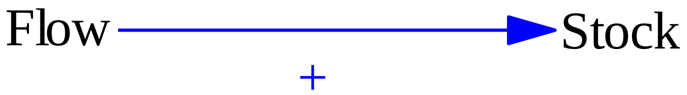
Example Model: Stocks

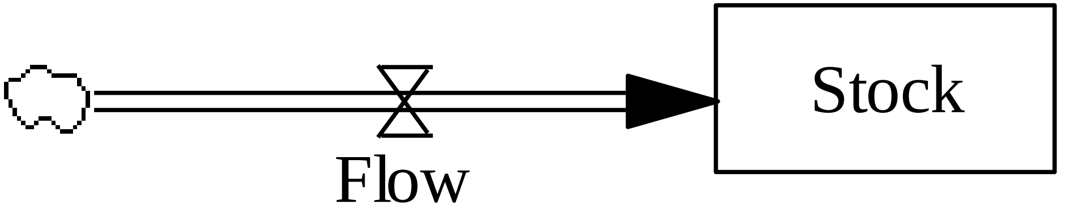


Example Model: Flows

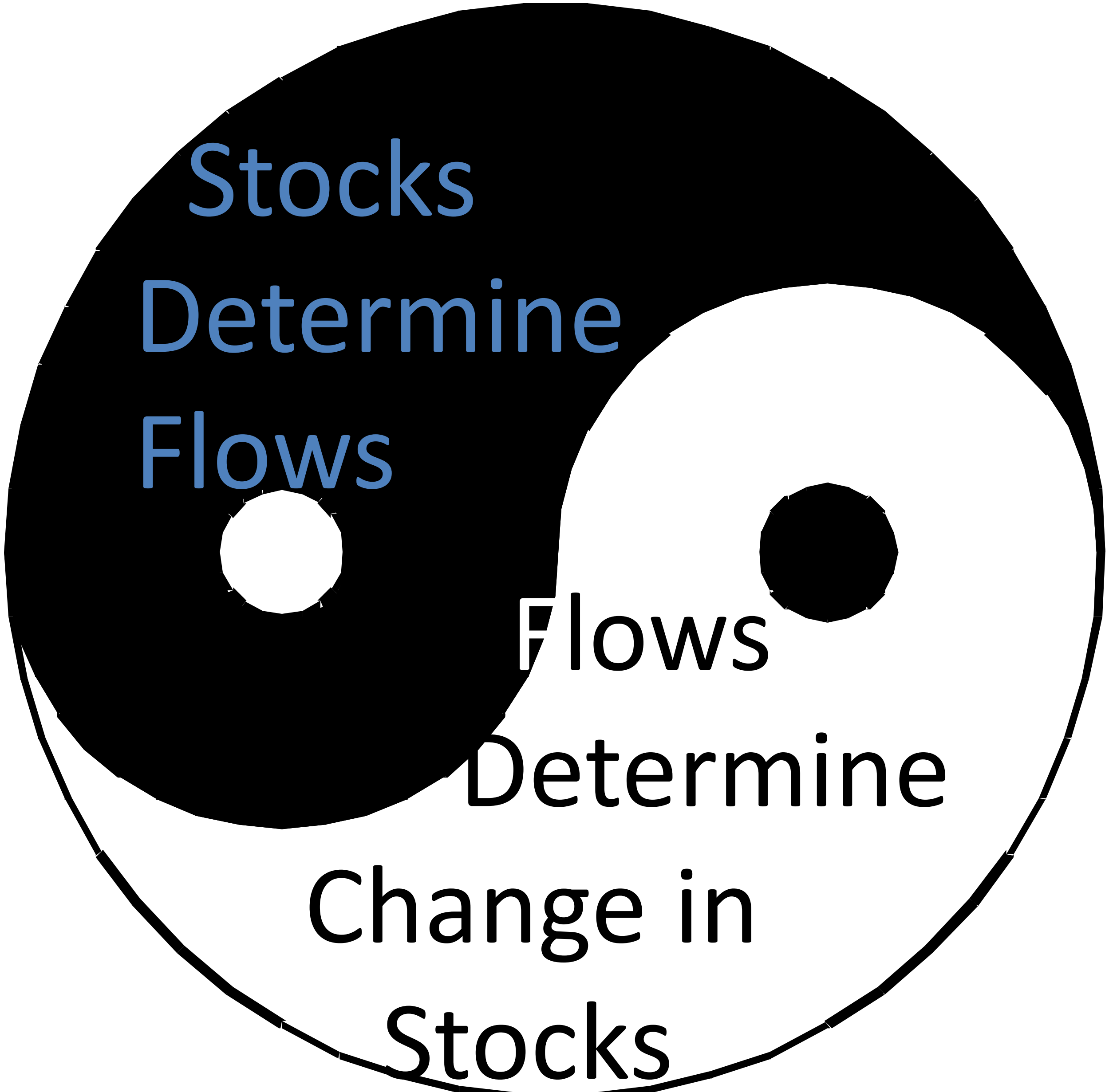


Key Component: Stock & Flow

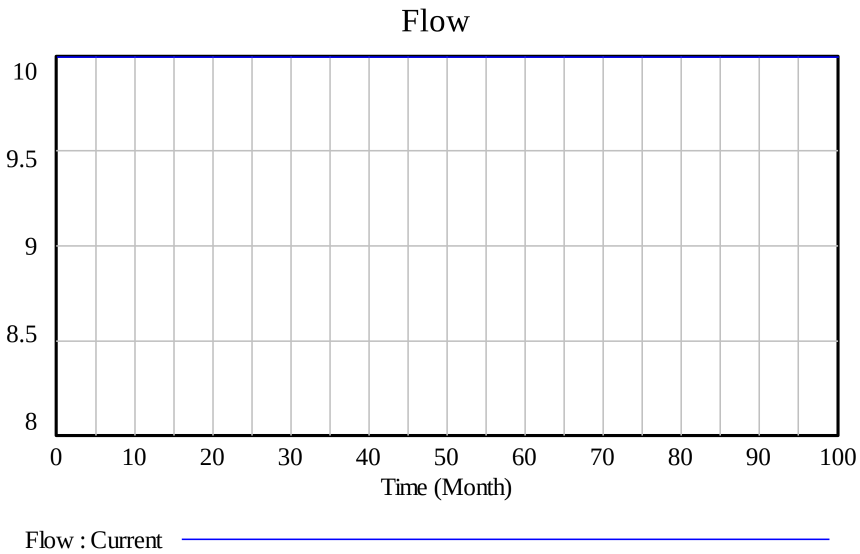


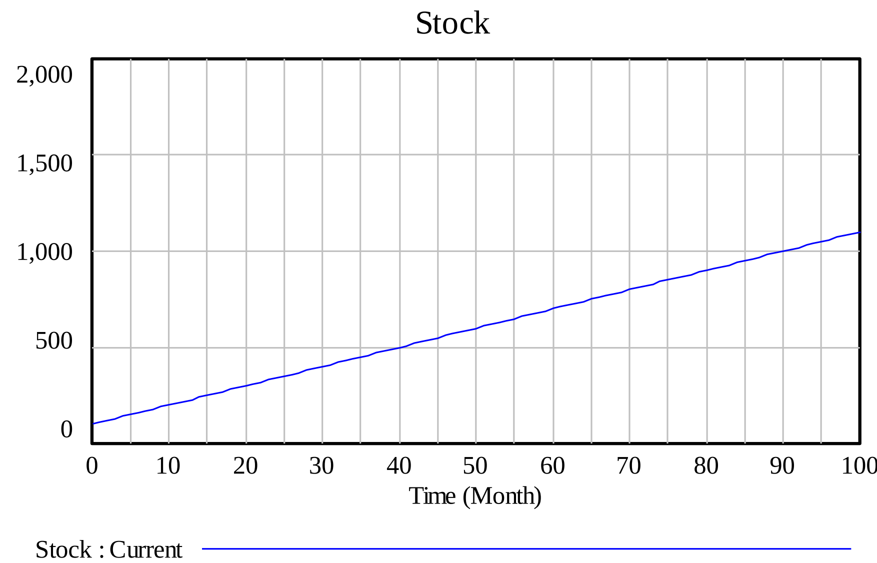


Structure & Behavior

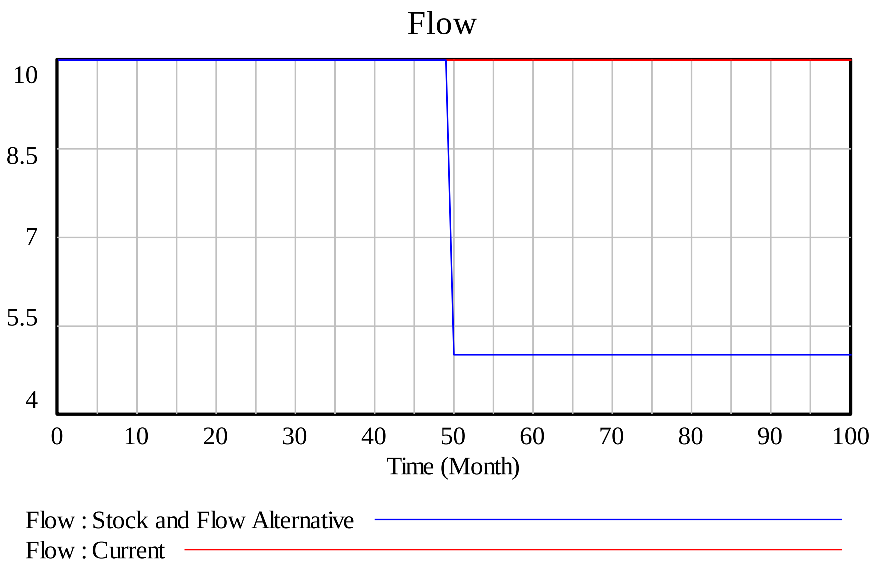


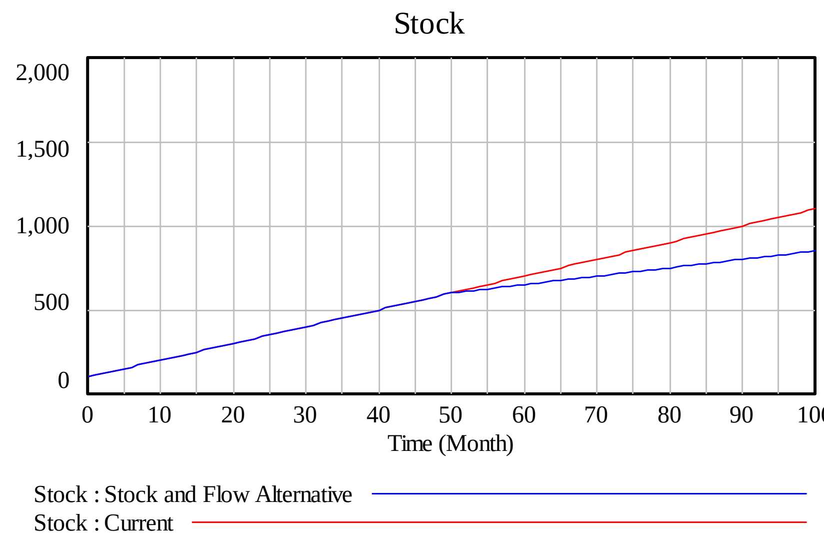
Net Flow Impact on Stock



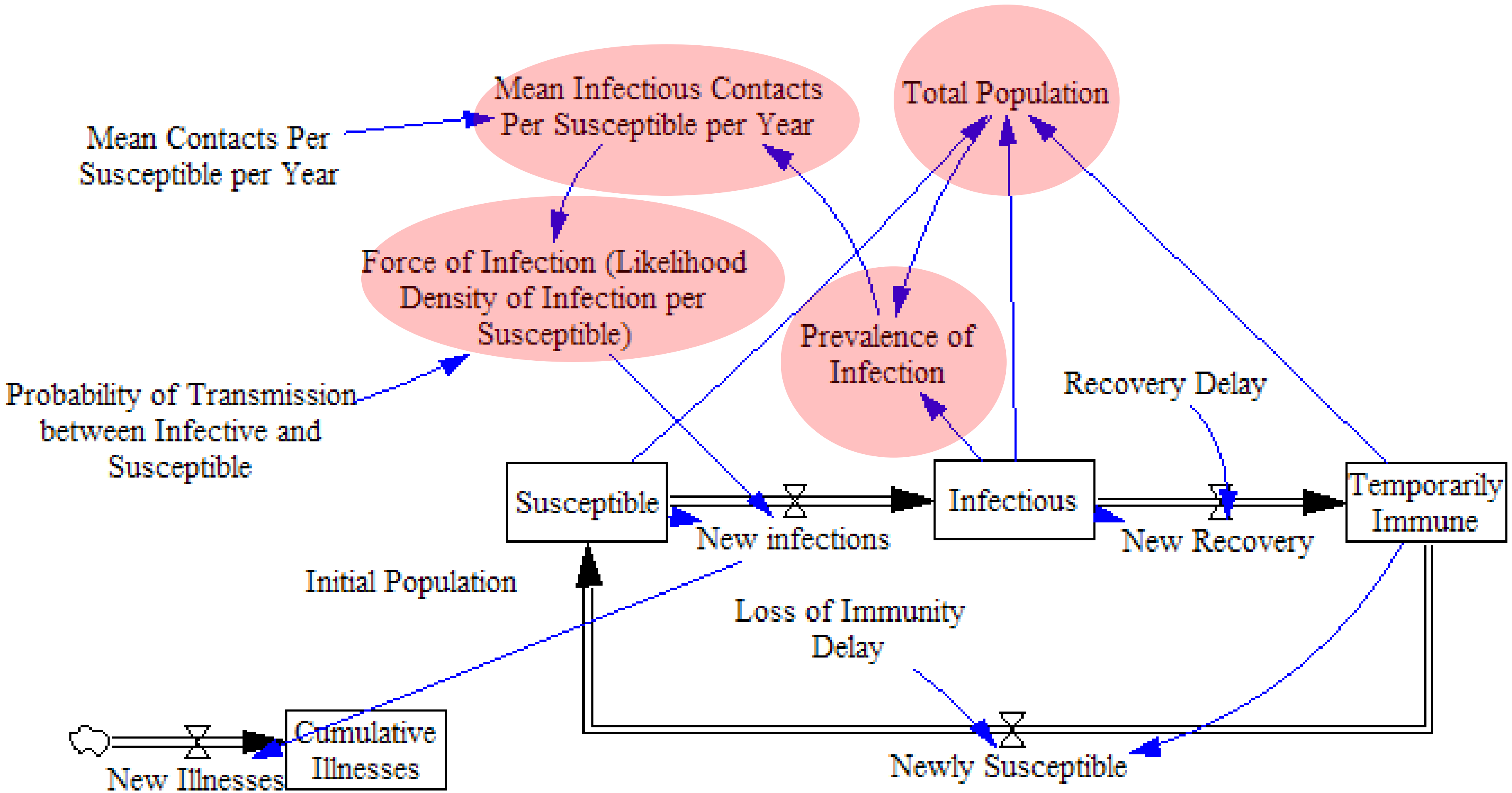


Impact of Lowering Flow (Rate) to 5/Month?





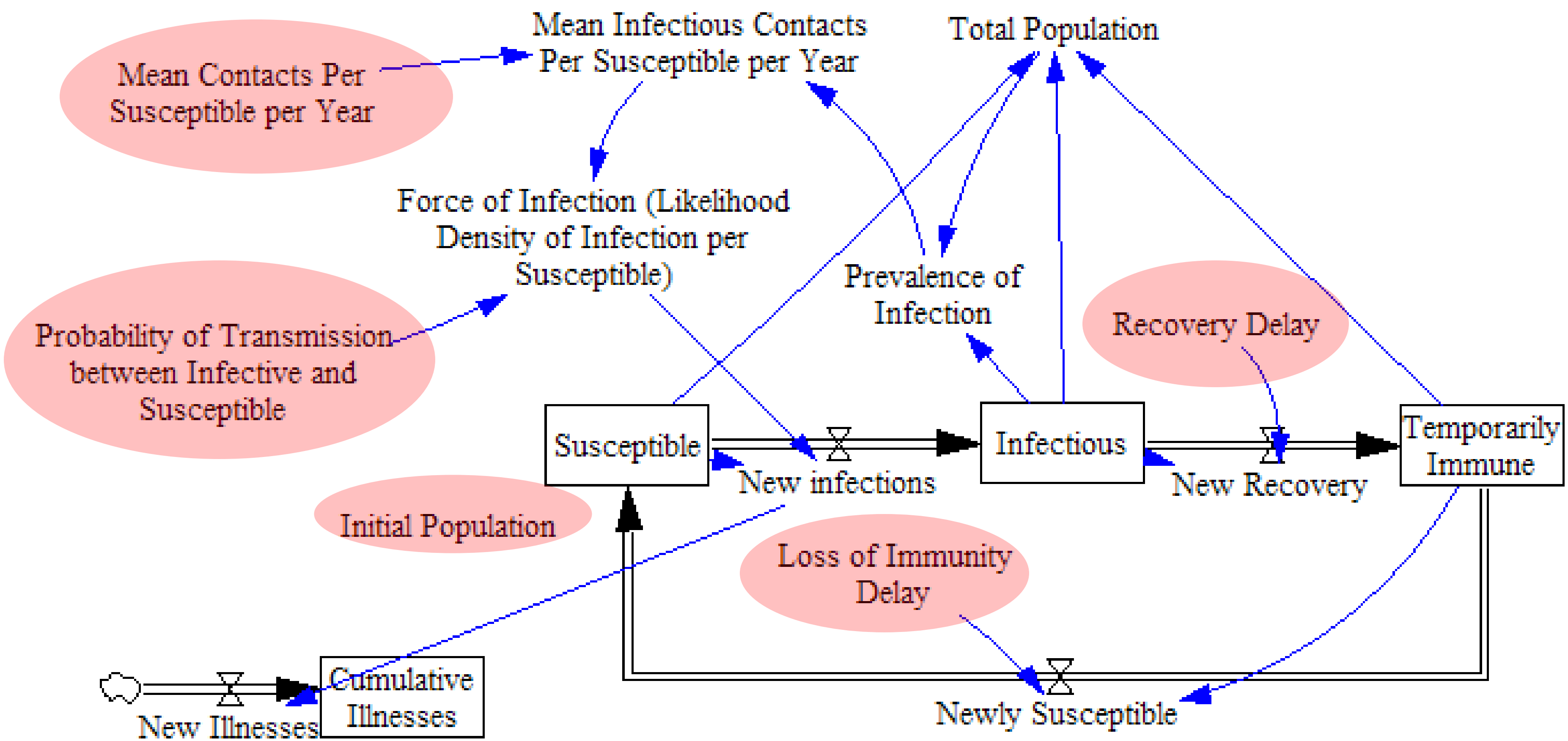
Example Model: Auxiliary Variables



Constants & Time Series Parameters

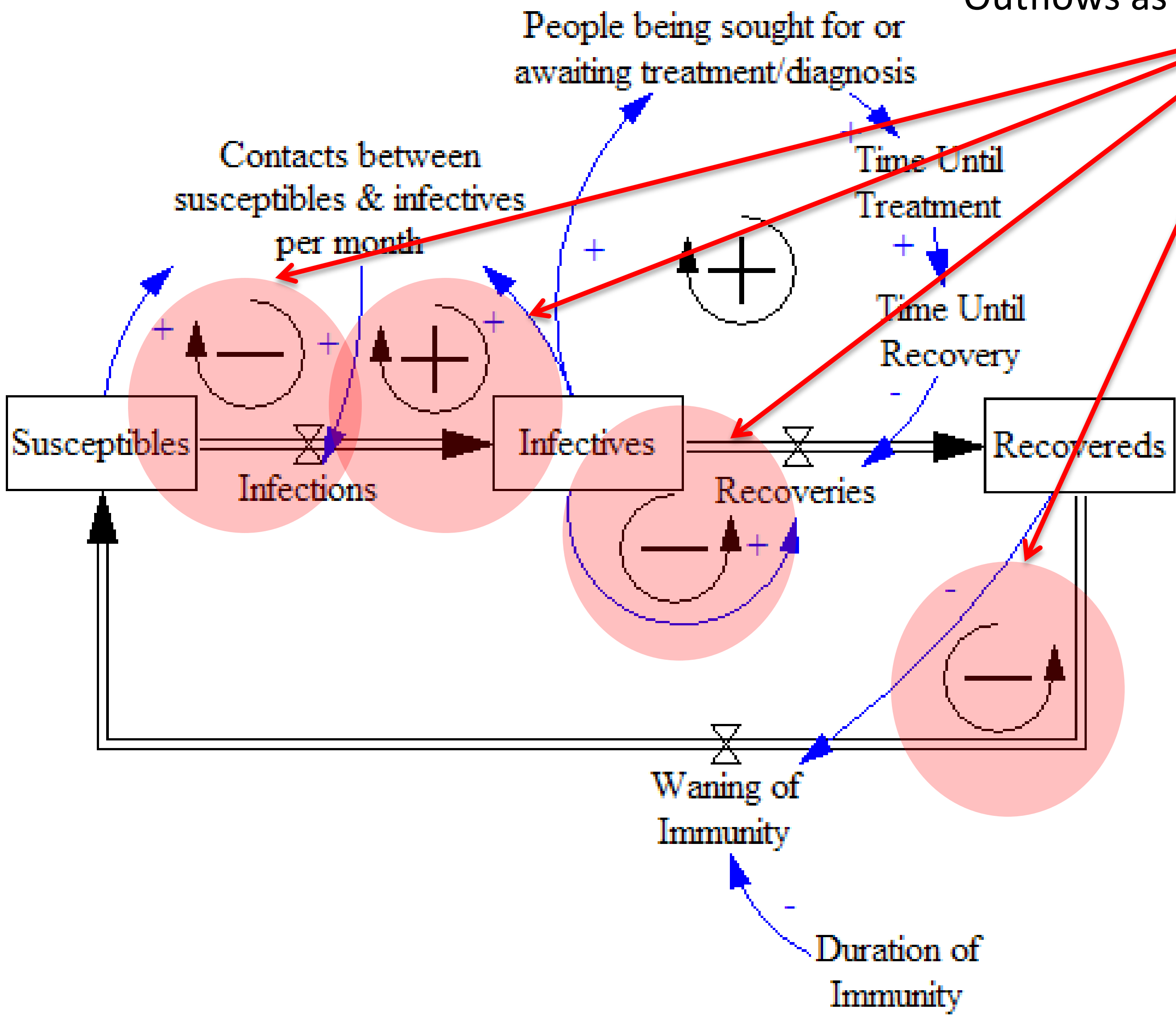
- For similar reasons to auxiliary variables, we give names to
  - Model constants
  - Time series

Example Model: Parameters



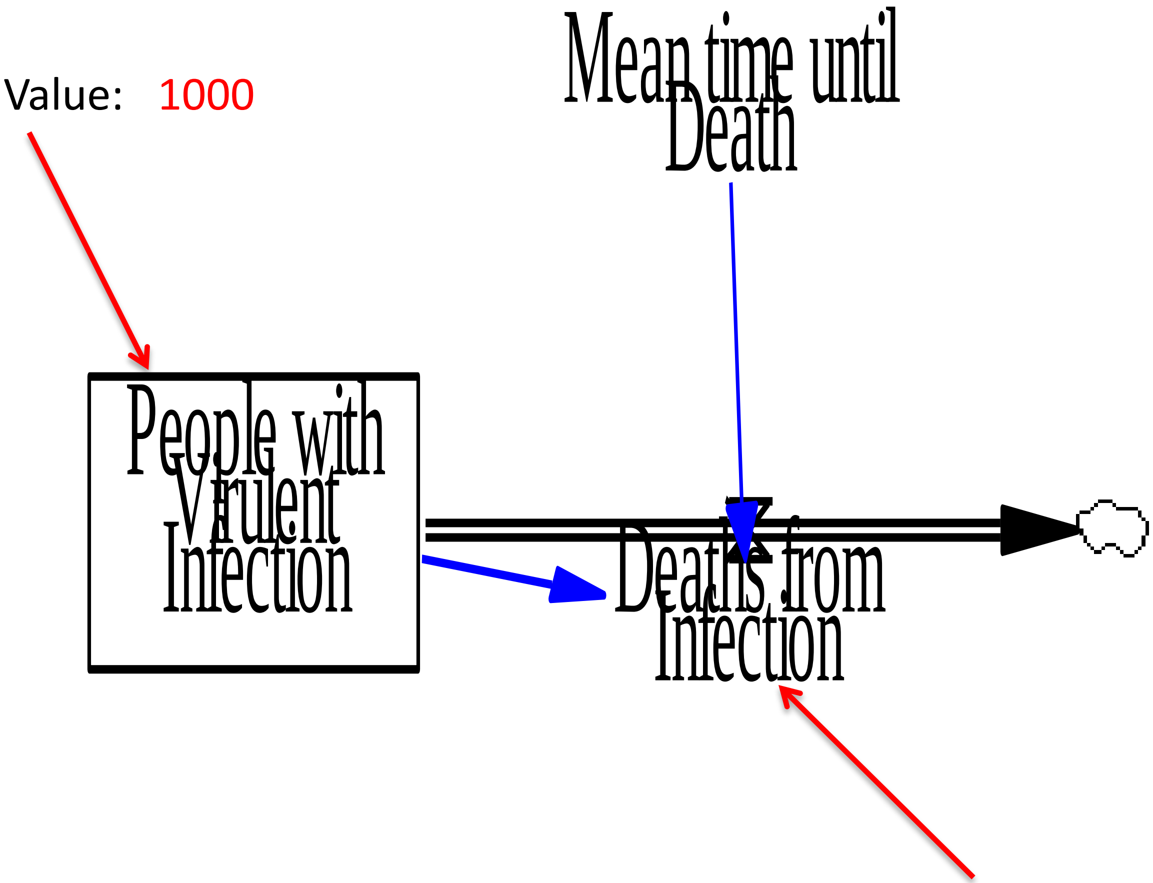
Example System Structure Diagram

Note treatment of flows as links from flow to stock
- Inflows as positive links
- Outflows as negative links



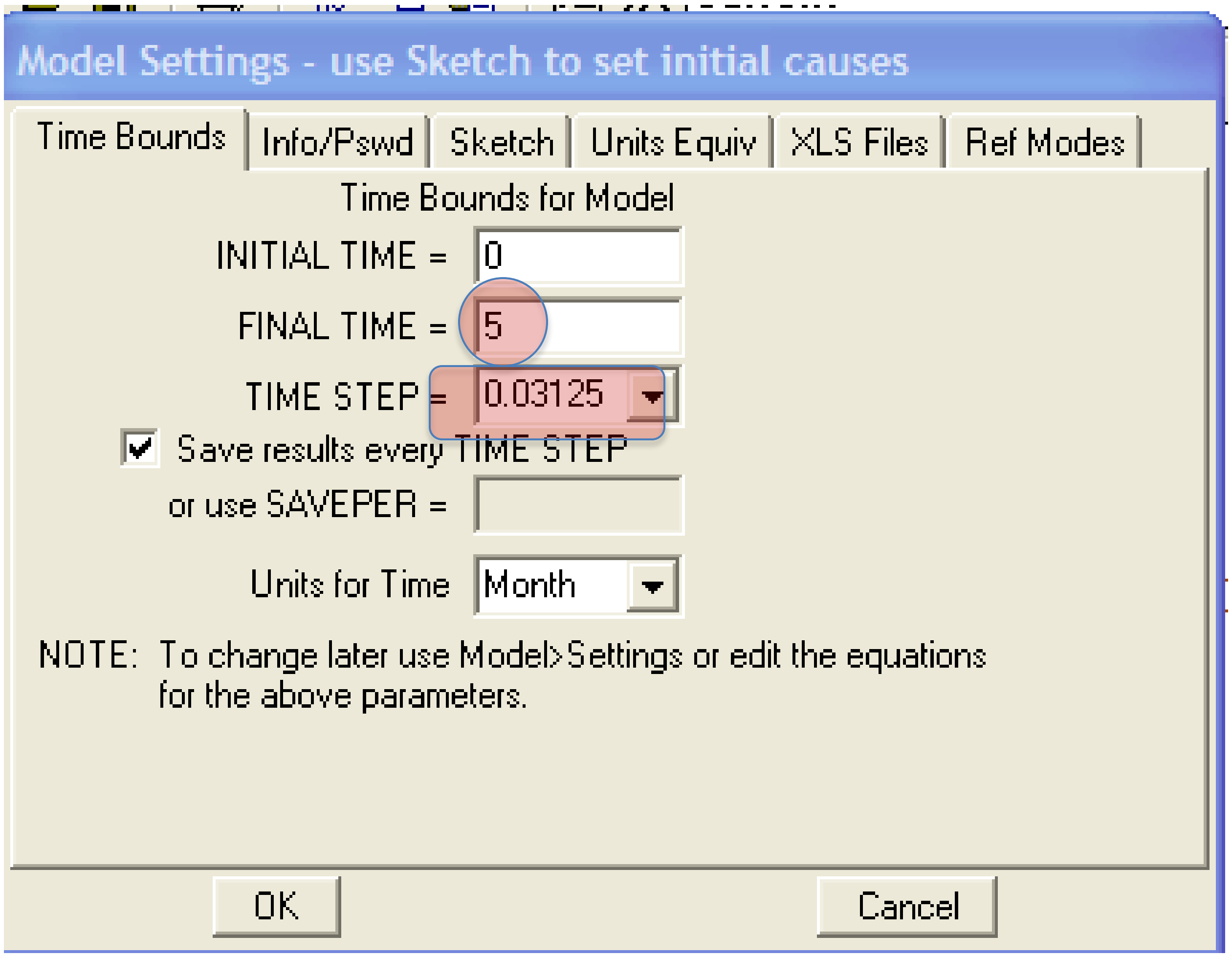
Simple First-Order Decay

Use Initial Value: 1000

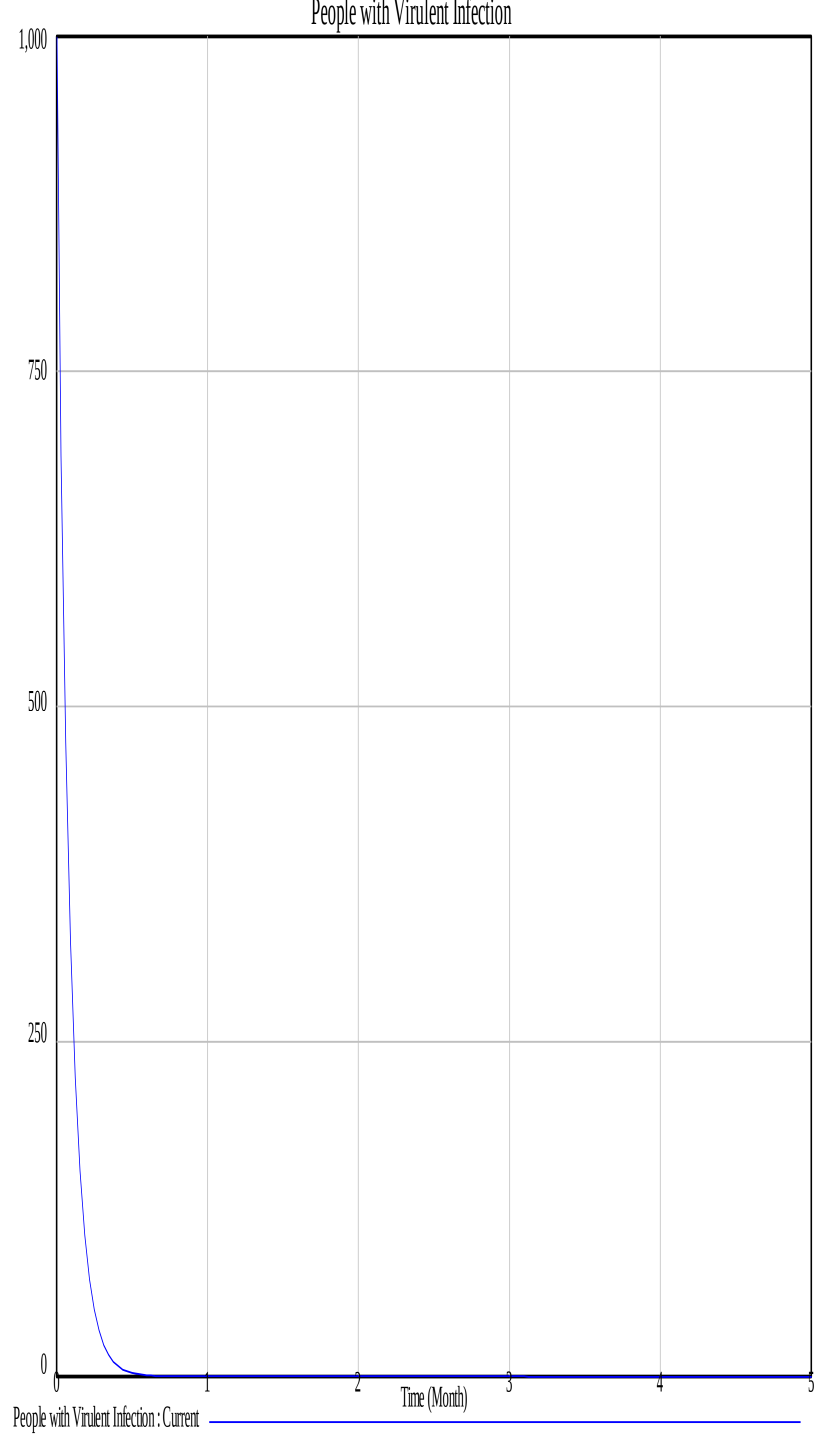


Use Formula: Deaths from Infection/Mean time until Death

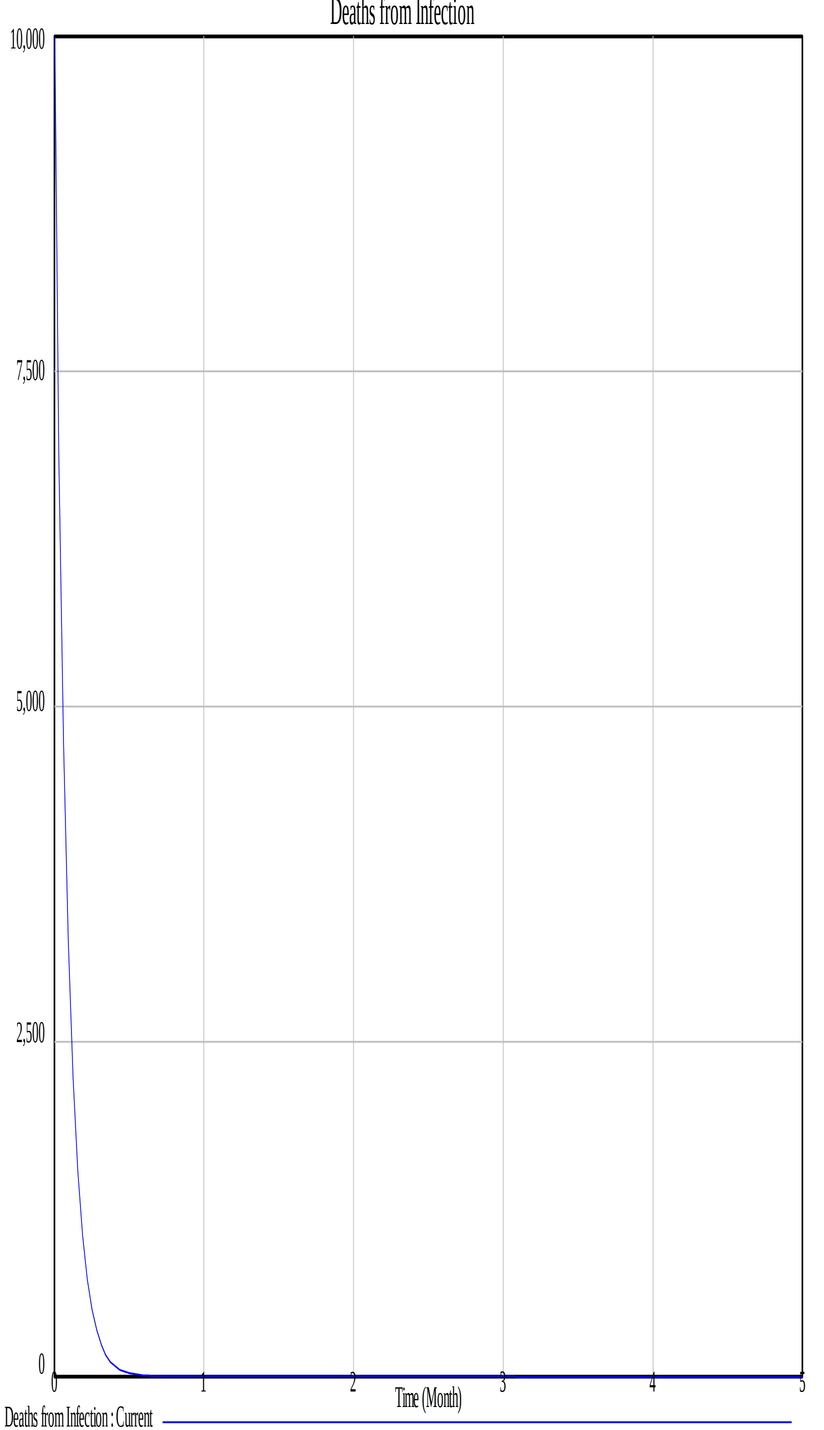
Set Model Settings (Model Menu/Settings Item)



Dynamics of Stock?



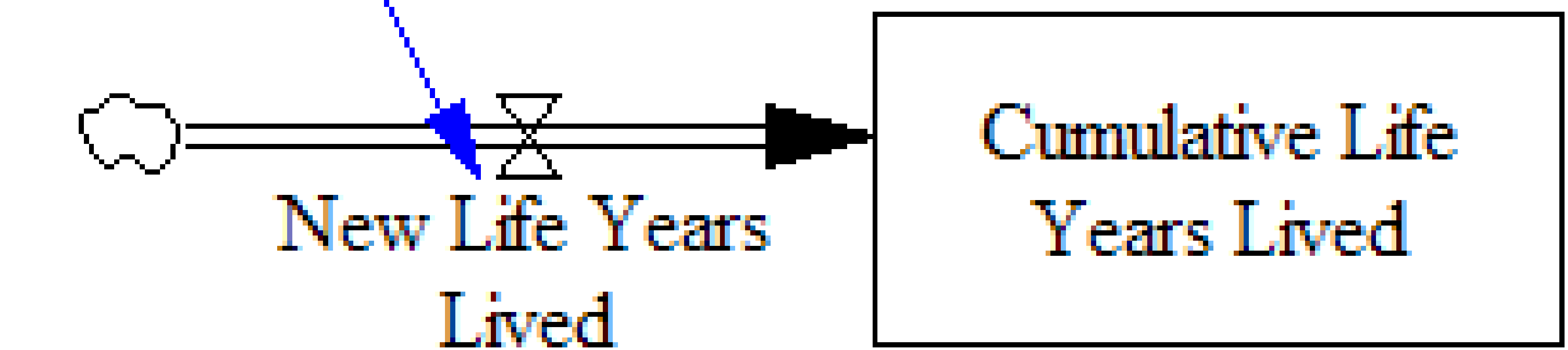
Dynamics of (Rate of) Death Flow?



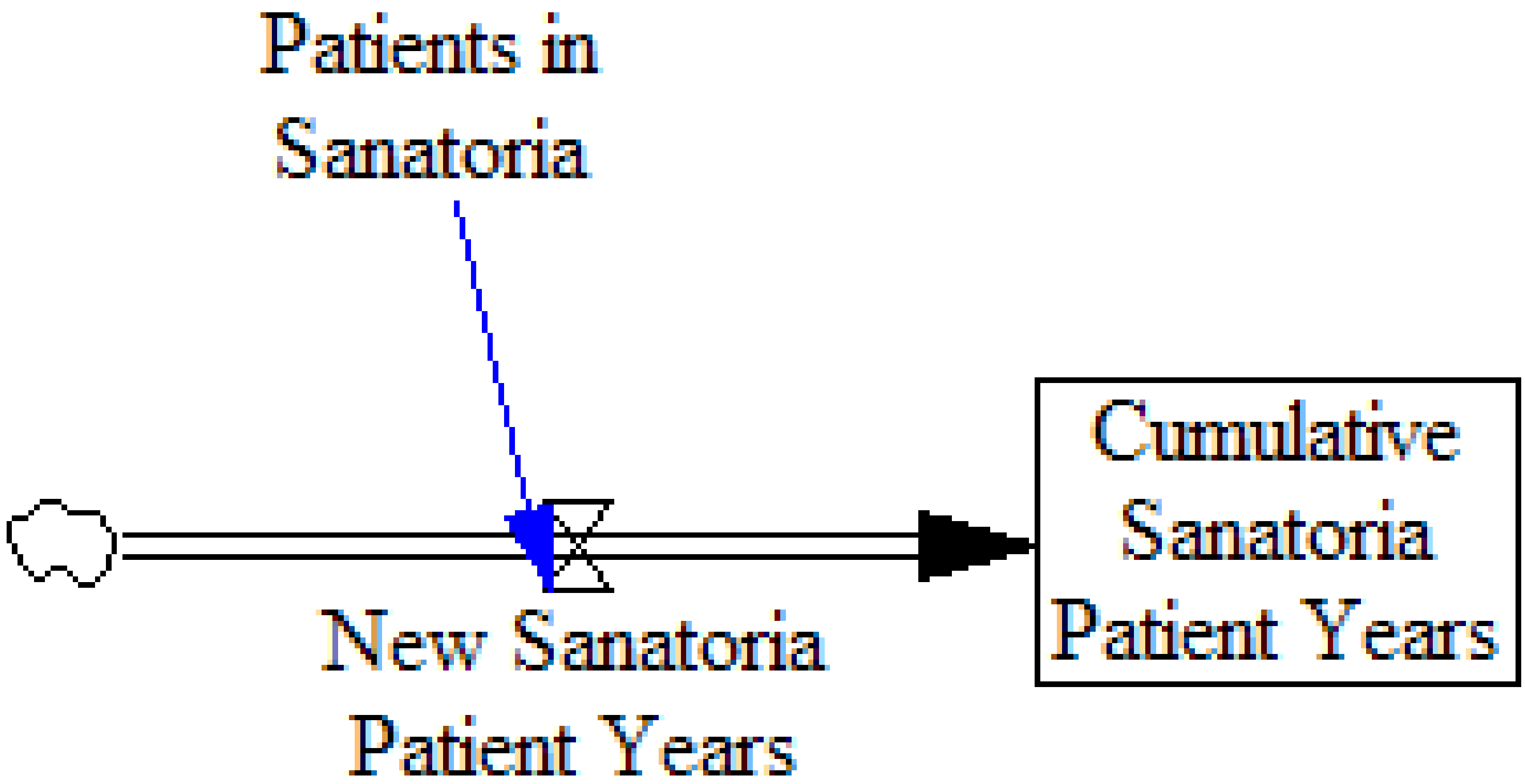
Stocks As Accumulations

- We often use stocks to accumulate (integrate) other (evolving) quantities over time
- Example (assume time measured in years):

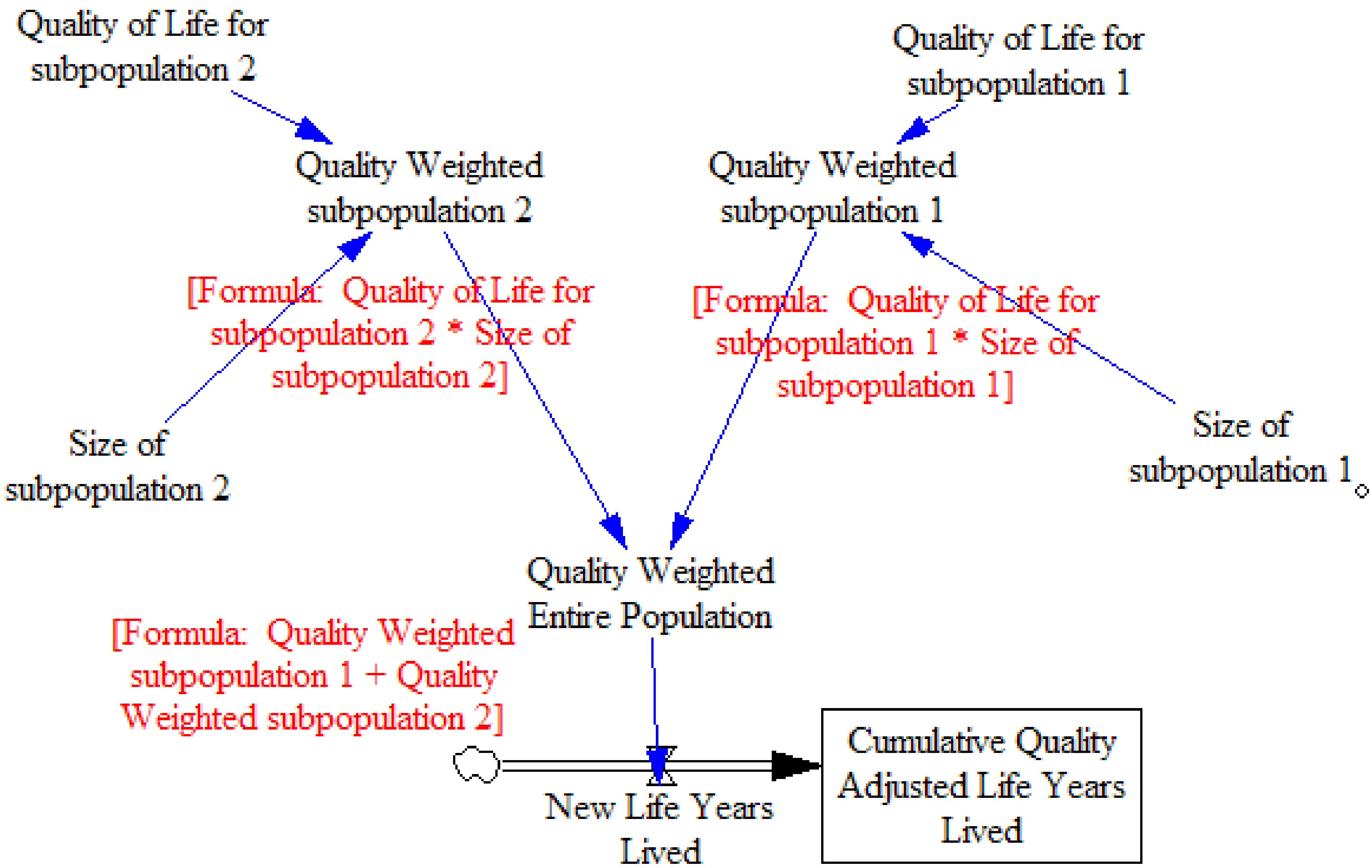
A Key Reflection: If we have population of size *P*, after 1 year, the stock holds 1**P*. After 2 years, 2*P, after *n* years, *n**P.
With a changing P, this integrates P over time.



Another Example of Stocks as Accumulations



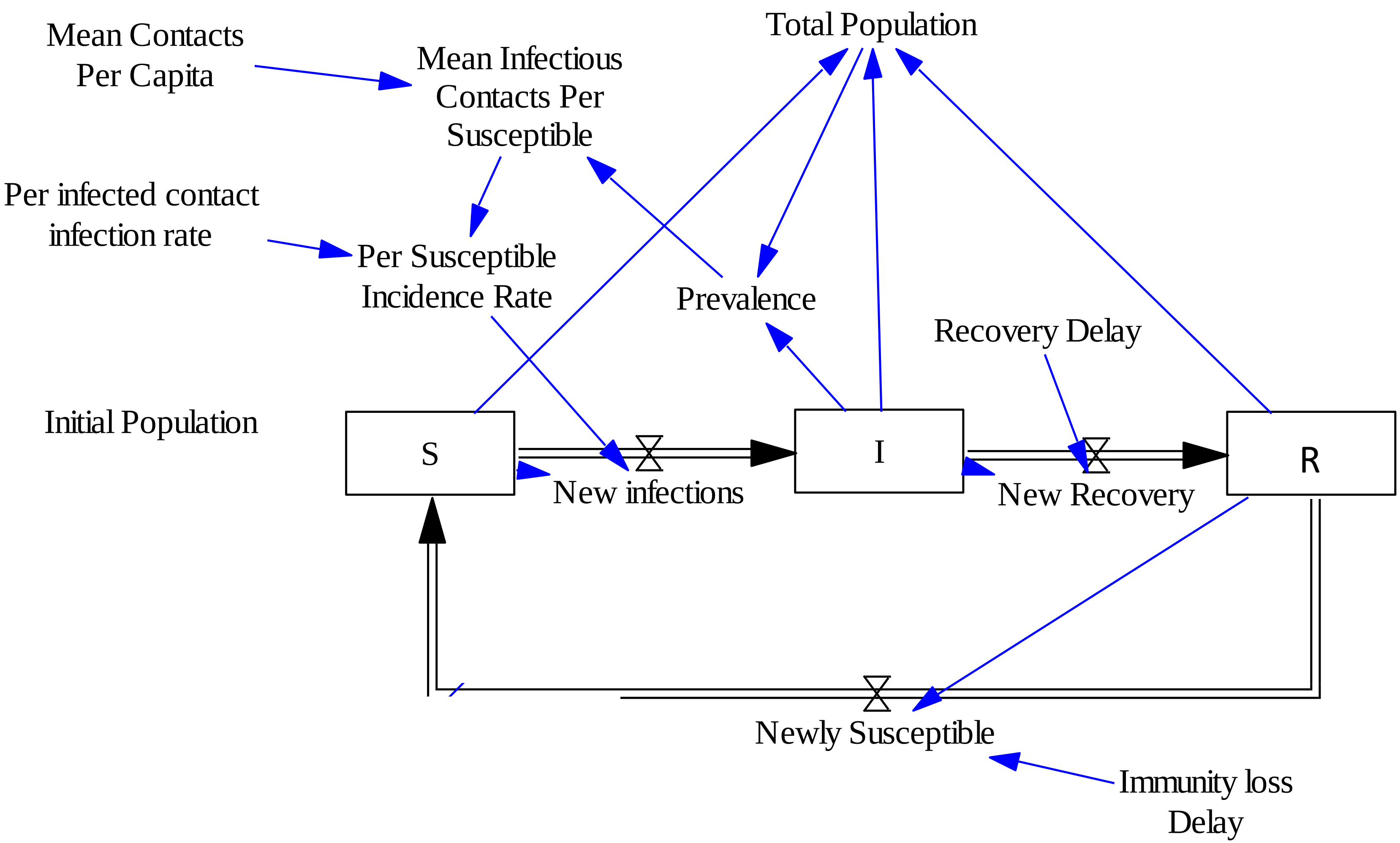
Slightly more Sophisticated



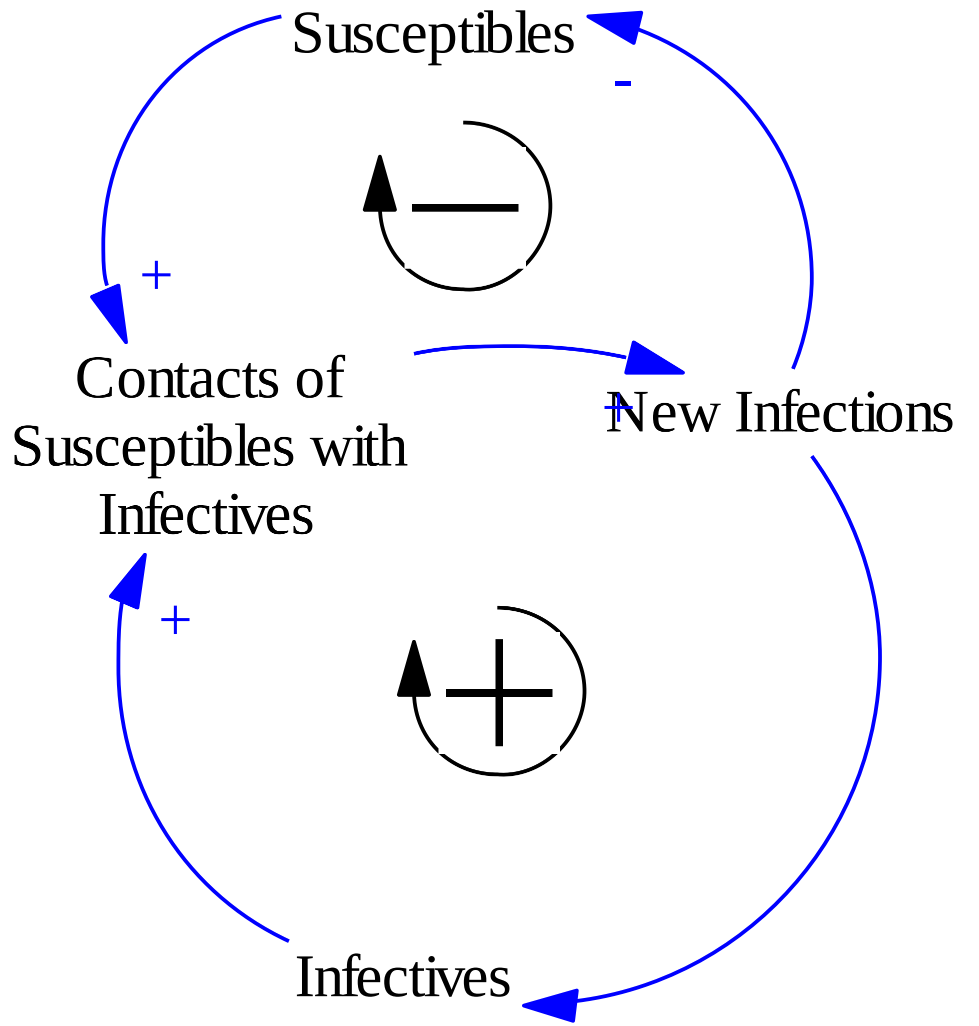
Principle: Structure Determines Behaviour

- Feedback & stock-and-flow structure of a system determines the possible patterns of behaviour
- Different sets of parameters (e.g. values for constants) will select particular behaviour within these behaviour patterns
- Changes to the feedback structure can change behaviour in fundamental ways

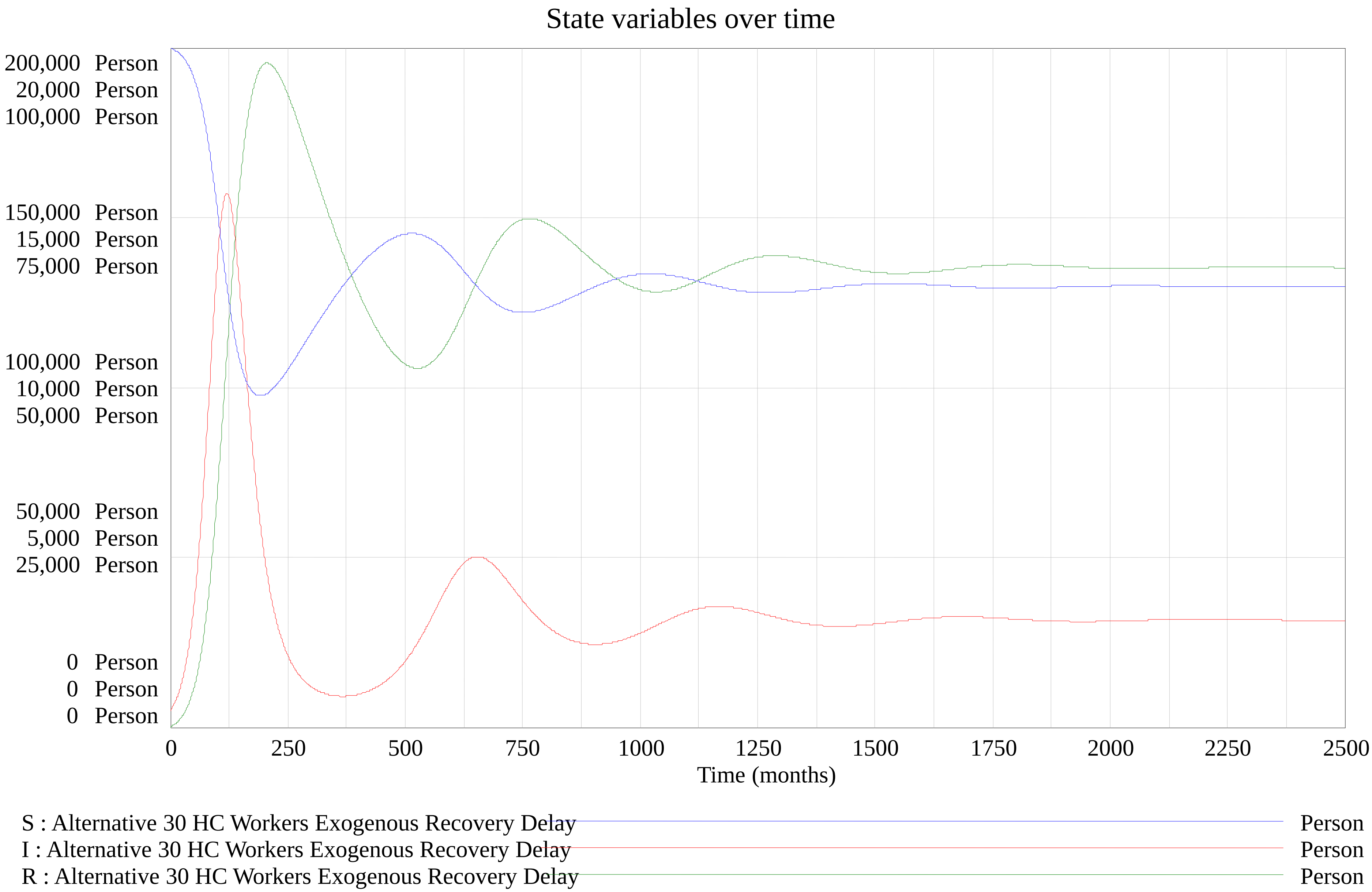
Simple SIT Model



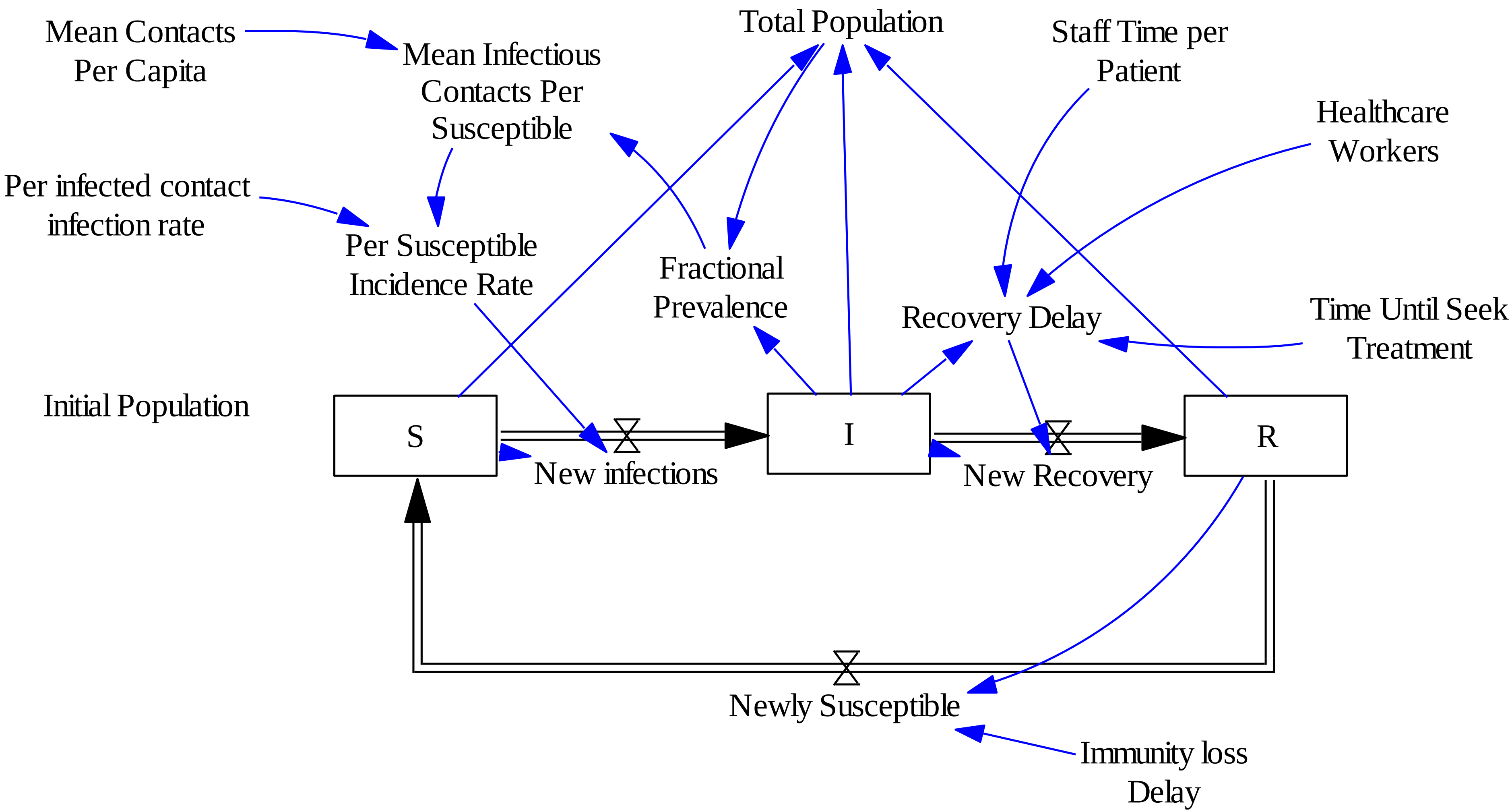
Classic Feedbacks



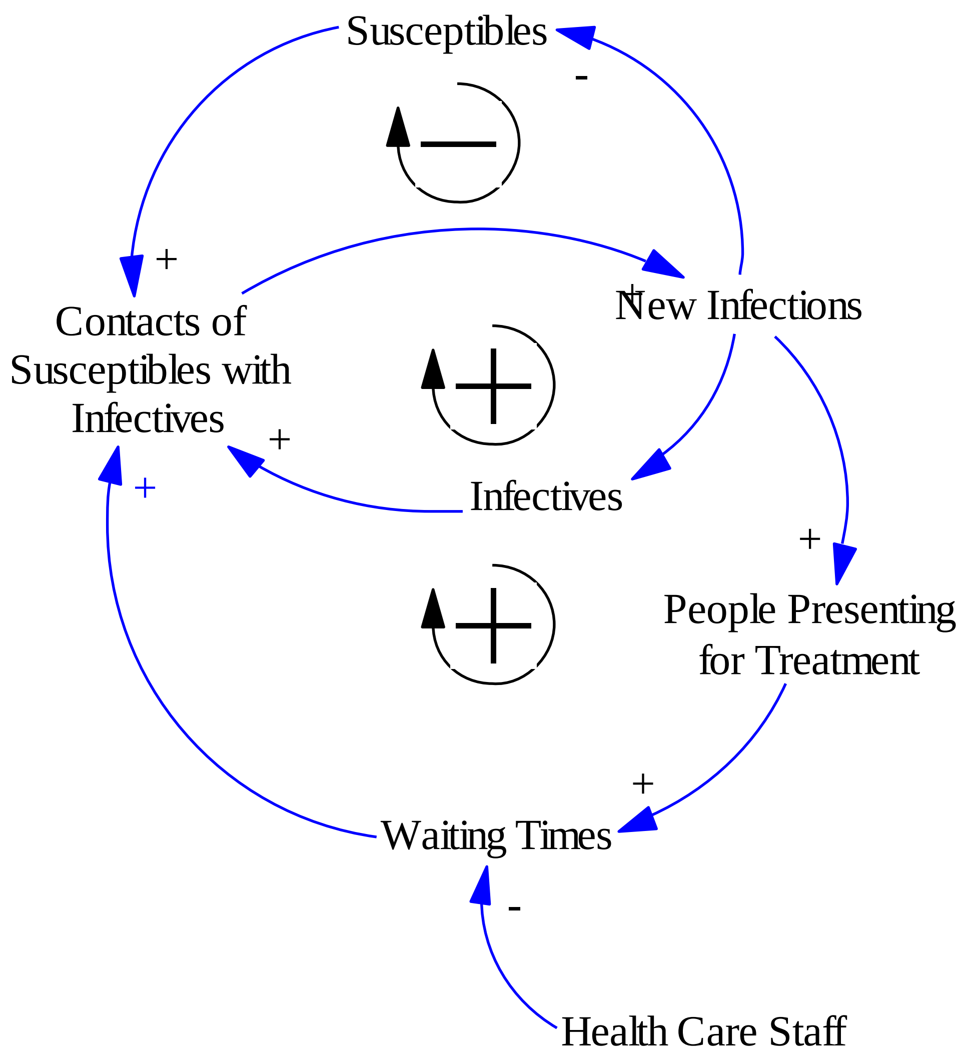
Dynamics


State variables over time
200,000 Person
20,000 Person
100,000 Person
150,000 Person
15,000 Person
75,000 Person
100,000 Person
10,000 Person
50,000 Person
50,000 Person
5,000 Person
25,000 Person
0 Person
0 Person
0 Person
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
Time (months)
S : Alternative 30 HC Workers Exogenous Recovery Delay
Person
I : Alternative 30 HC Workers Exogenous Recovery Delay
Person
R : Alternative 30 HC Workers Exogenous Recovery Delay
Person

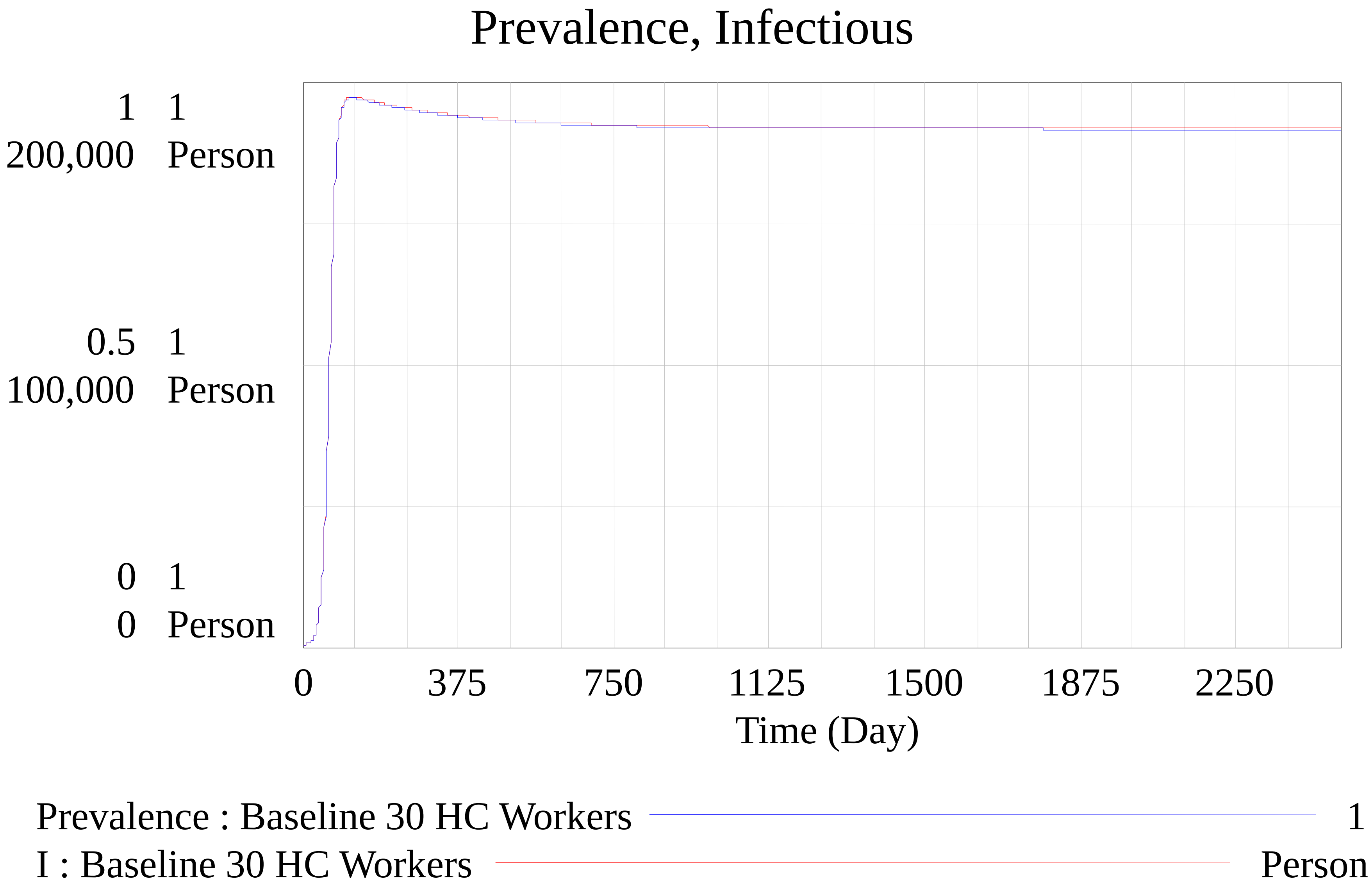
Broadening the Model Boundaries: Endogenous Recovery Delay



Broadening the Model Boundaries: Endogenous Recovery Delay



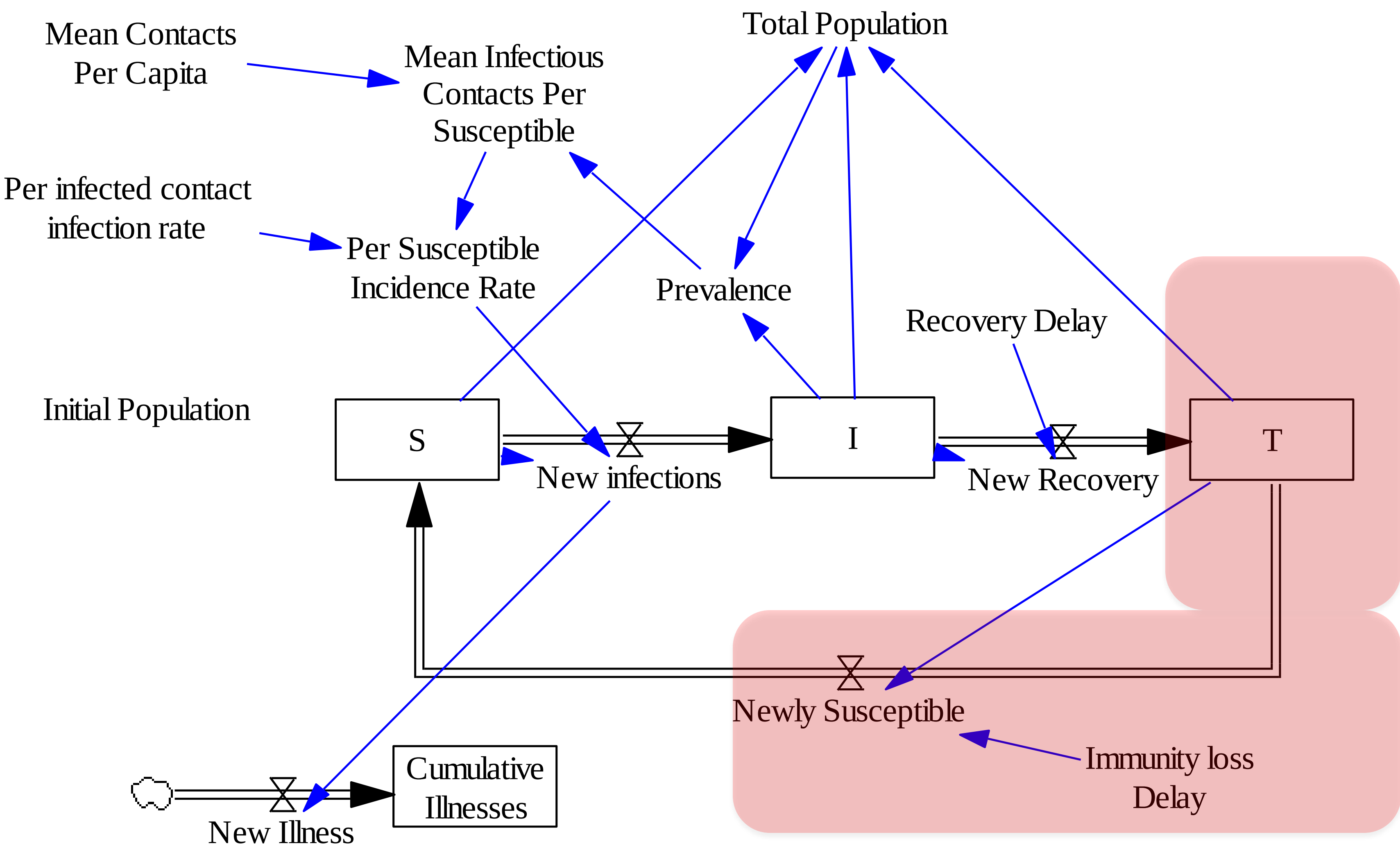
A Different Behaviour Mode



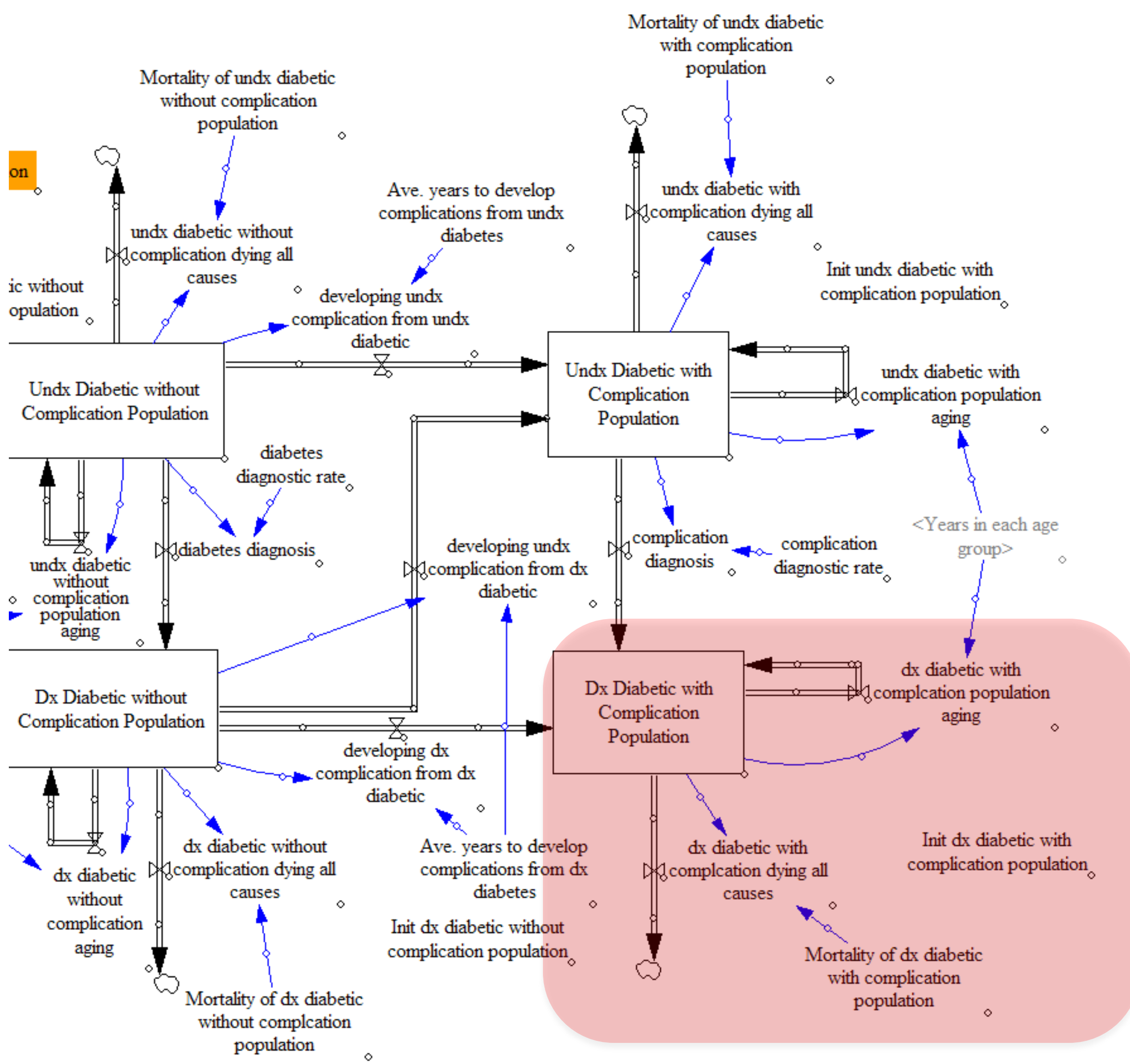
Structure as Shaping Behaviour

- System structure is defined by
  - Stocks
  - Flows
  - Connections between them
- Nonlinearity: The behaviour of the whole is more than the sum of the behaviour of the parts
  - "Emergent" behaviour would not be anticipated from simple behaviour of each piece in turn
- Stock and flow structure (including feedbacks) of a system determines the qualitative behaviour modes that the system can take on

First Order Delays in Action: Simple SIT Model



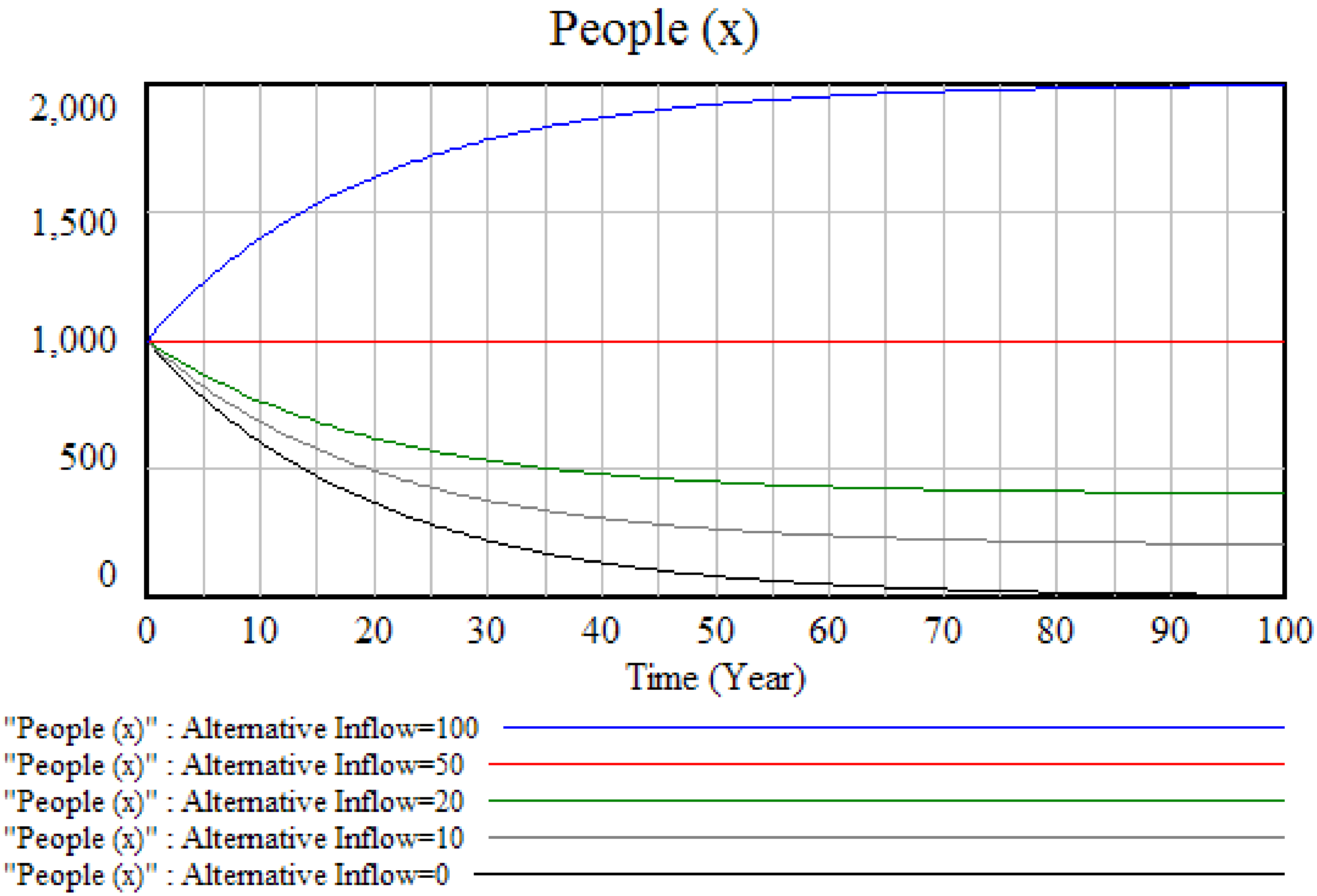
First Order Delays in Action: Simple SIT Model



Scenarios for First Order Delay: Variation in Inflow Rates

- For different immigration (inflows) (what do you expect?)
  - Inflow=10
  - Inflow=20
  - Inflow=50
  - Inflow=100
  - Why do you see this “goal seeking” pattern?
  - What is the “goal” being sought?

Behaviour of Stock for Different Inflows

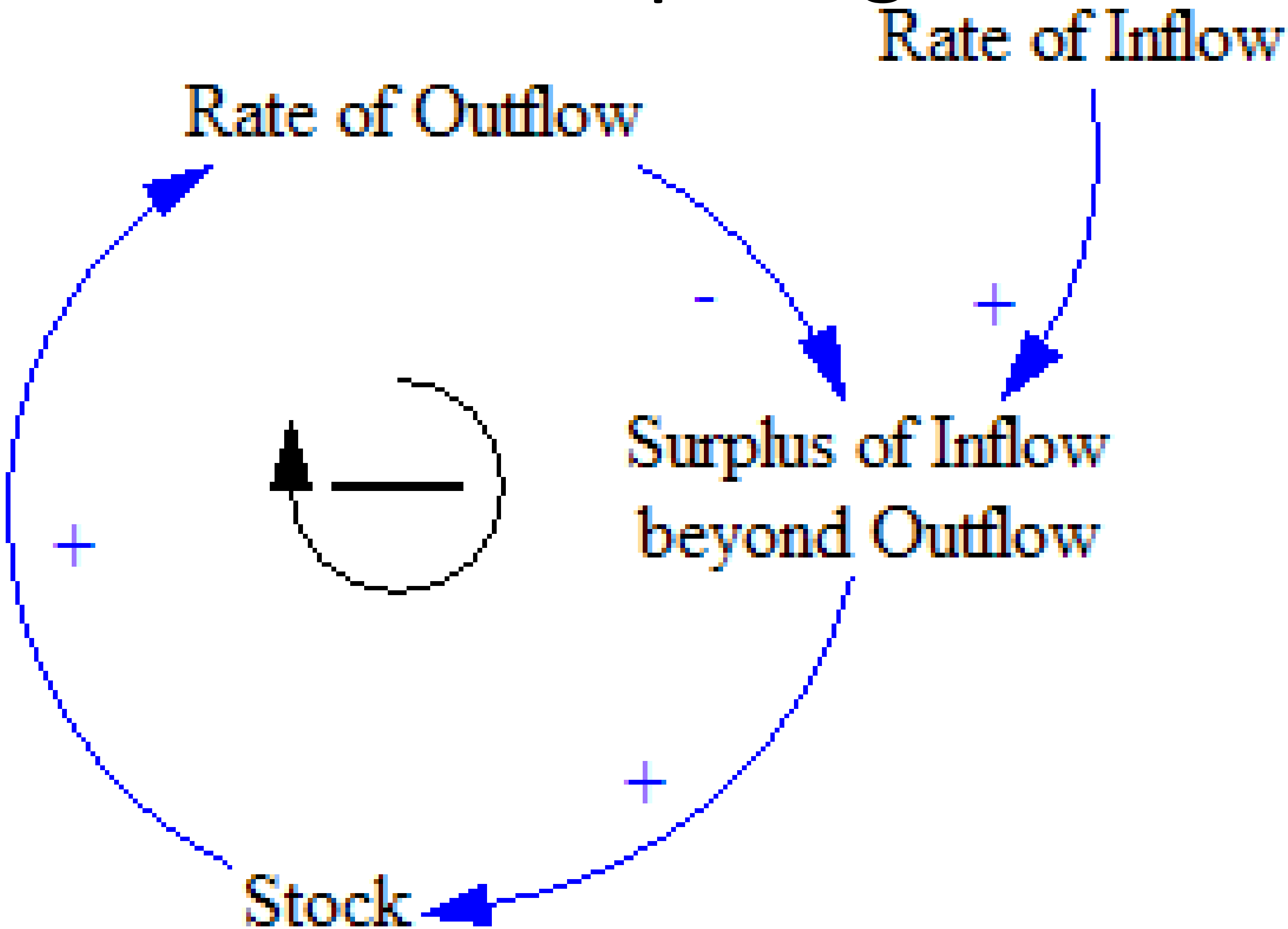


Why do we see this behaviour?

Goal Seeking Behaviour

- The goal seeking behaviour is associated with a negative feedback loop
  - The larger the population in the stock, the more people die per year
- If we have more people coming in than are going out per year, the stock (and, hence, outflow!) rises until the point where inflow=outflows
- If we have fewer people coming in than are going out per year, the stock declines (& outflow) declines until the point where inflow=outflows

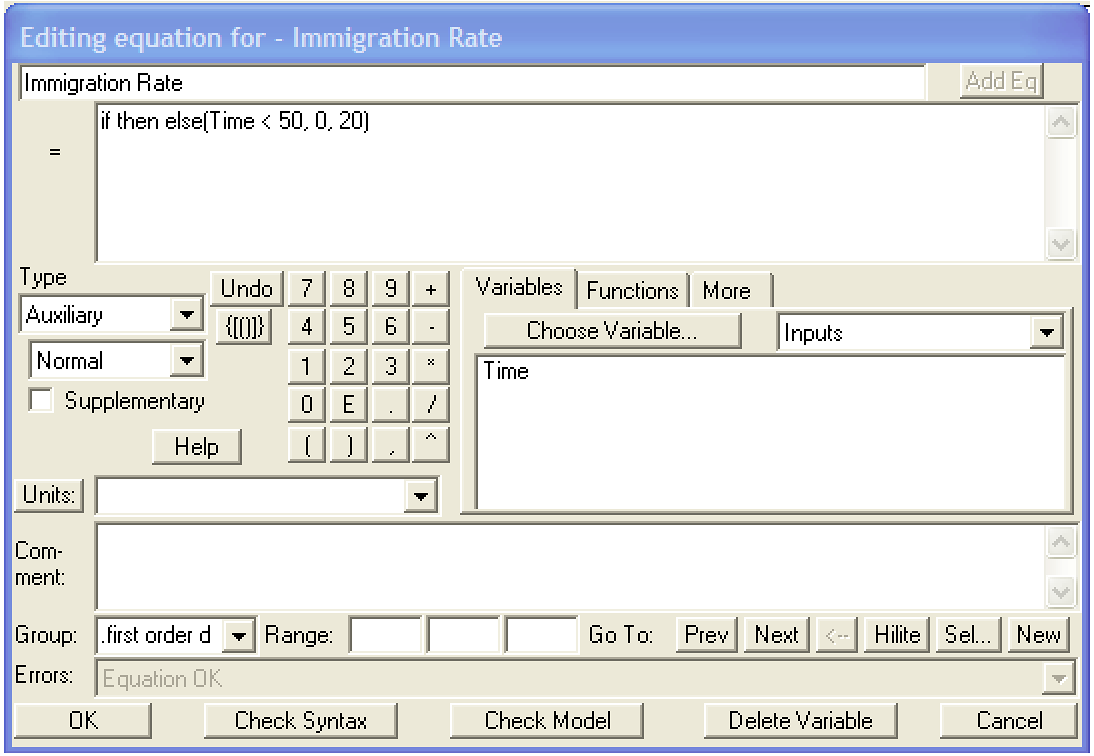
As a Causal Loop Diagram



What does this tell us about how the system would respond to a sudden change in immigration?

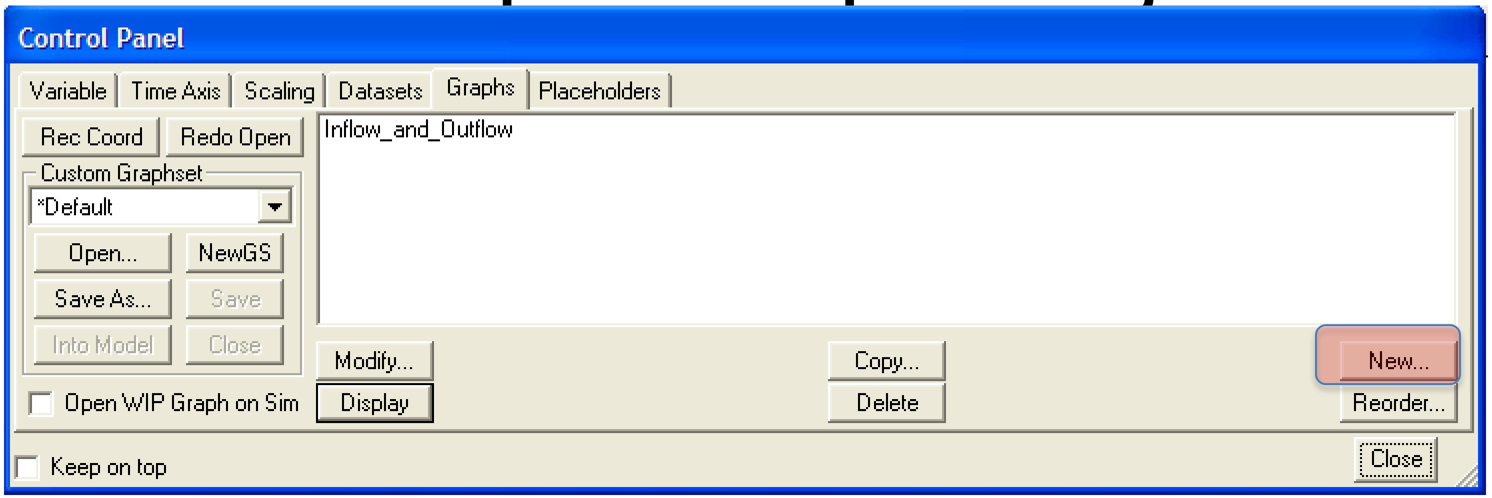
Response to a Change

- Feed in an immigration "step function" that rises suddenly from 0 to 20 at time 50

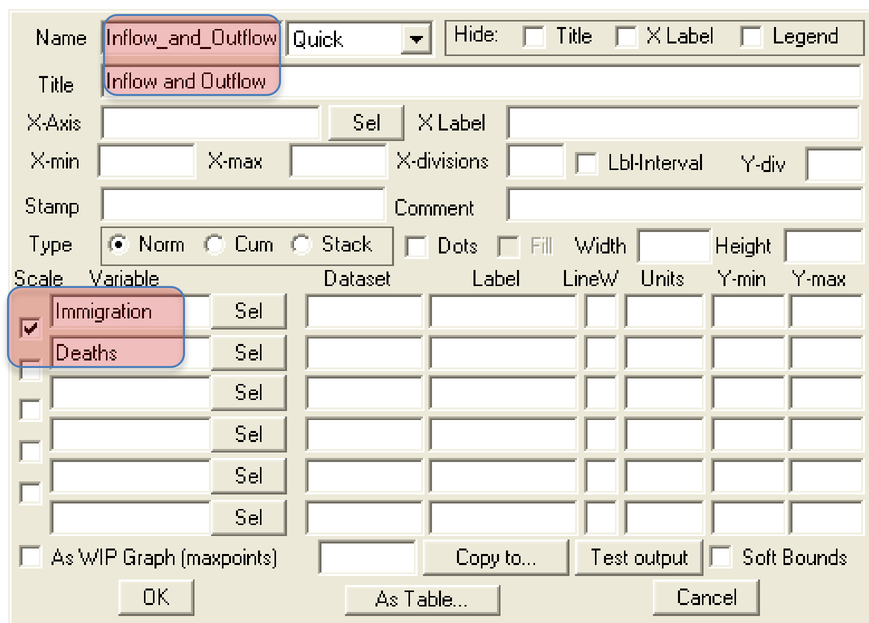


- Set the Initial Value of Stock to 0
- How does the stock change over time?

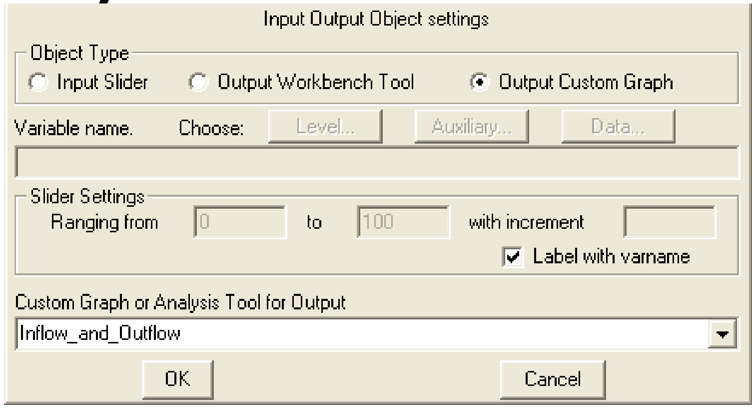
Create a Custom Graph & Display it as an Input-Output Object

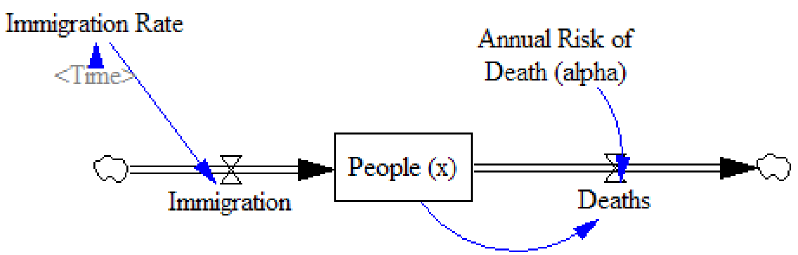


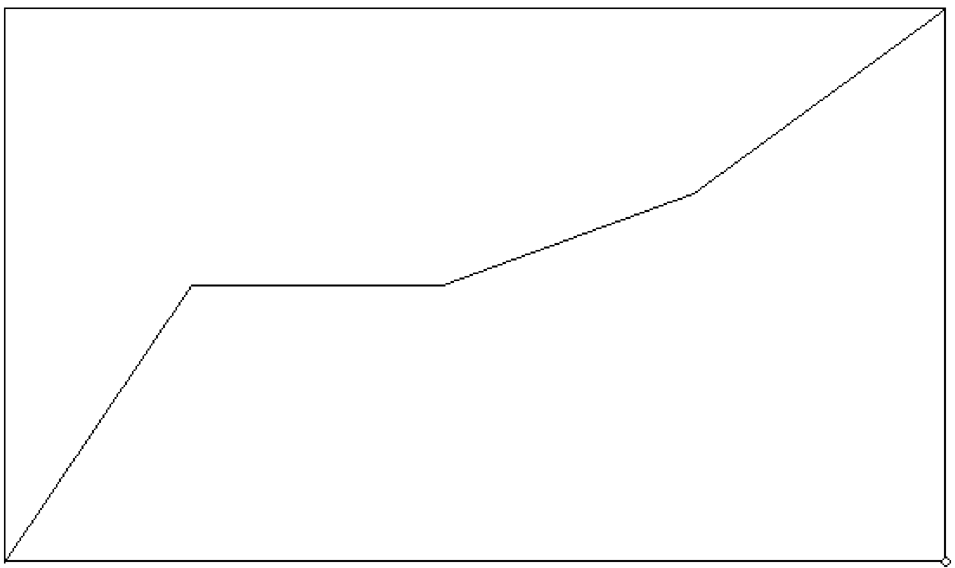
- Editing



Create Input-Output Object (for Synthesim)

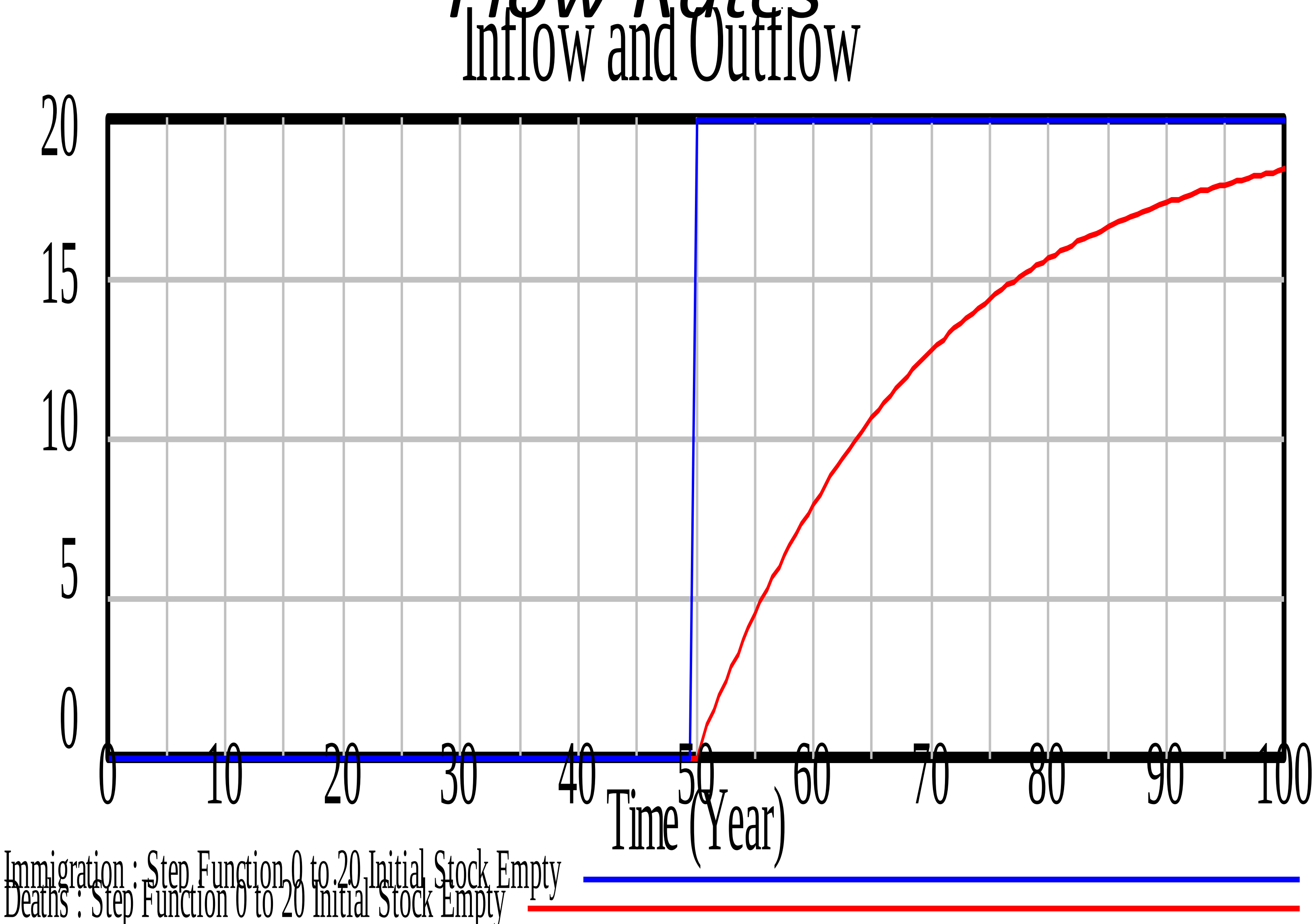






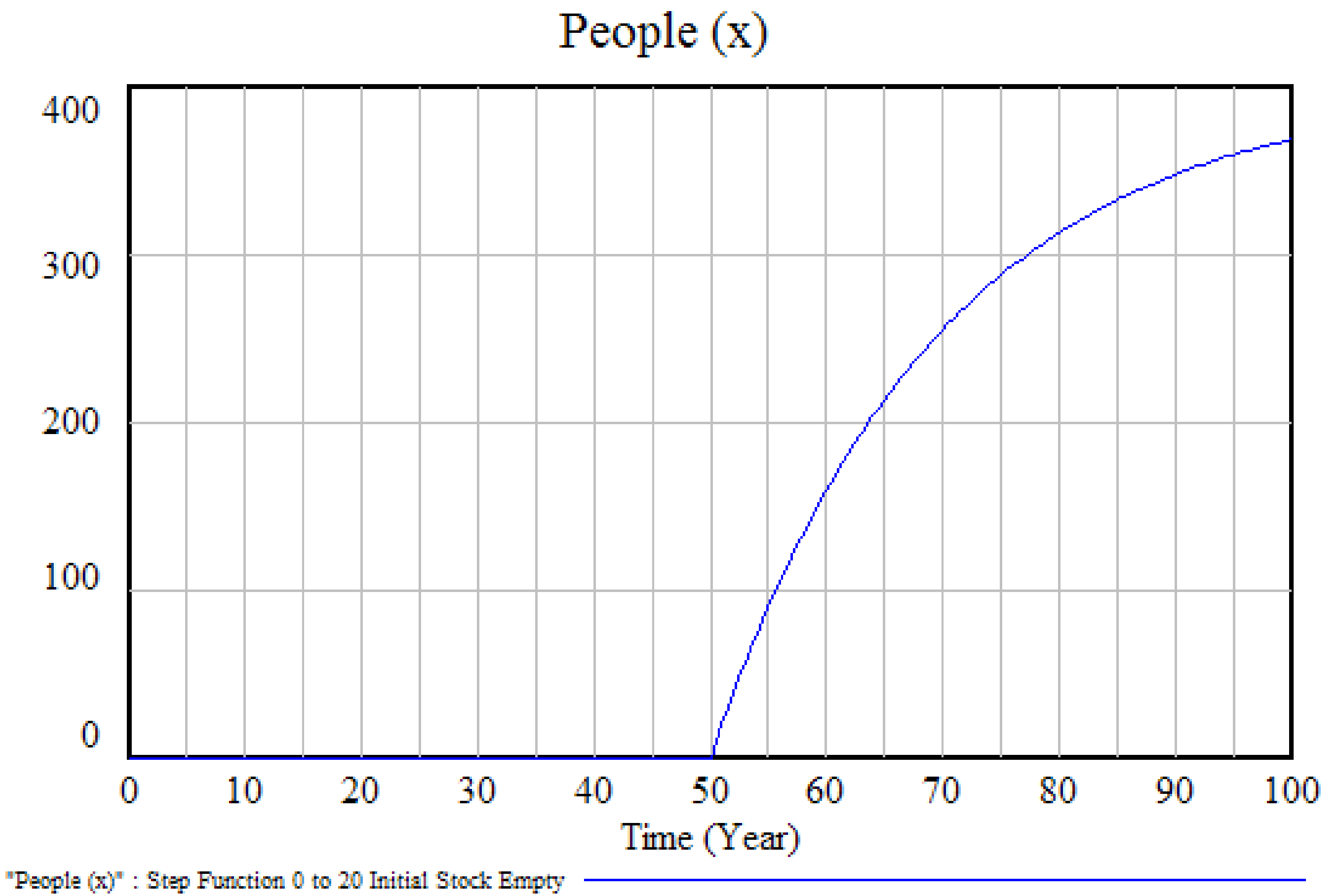
Stock Starting Empty

Flow Rates



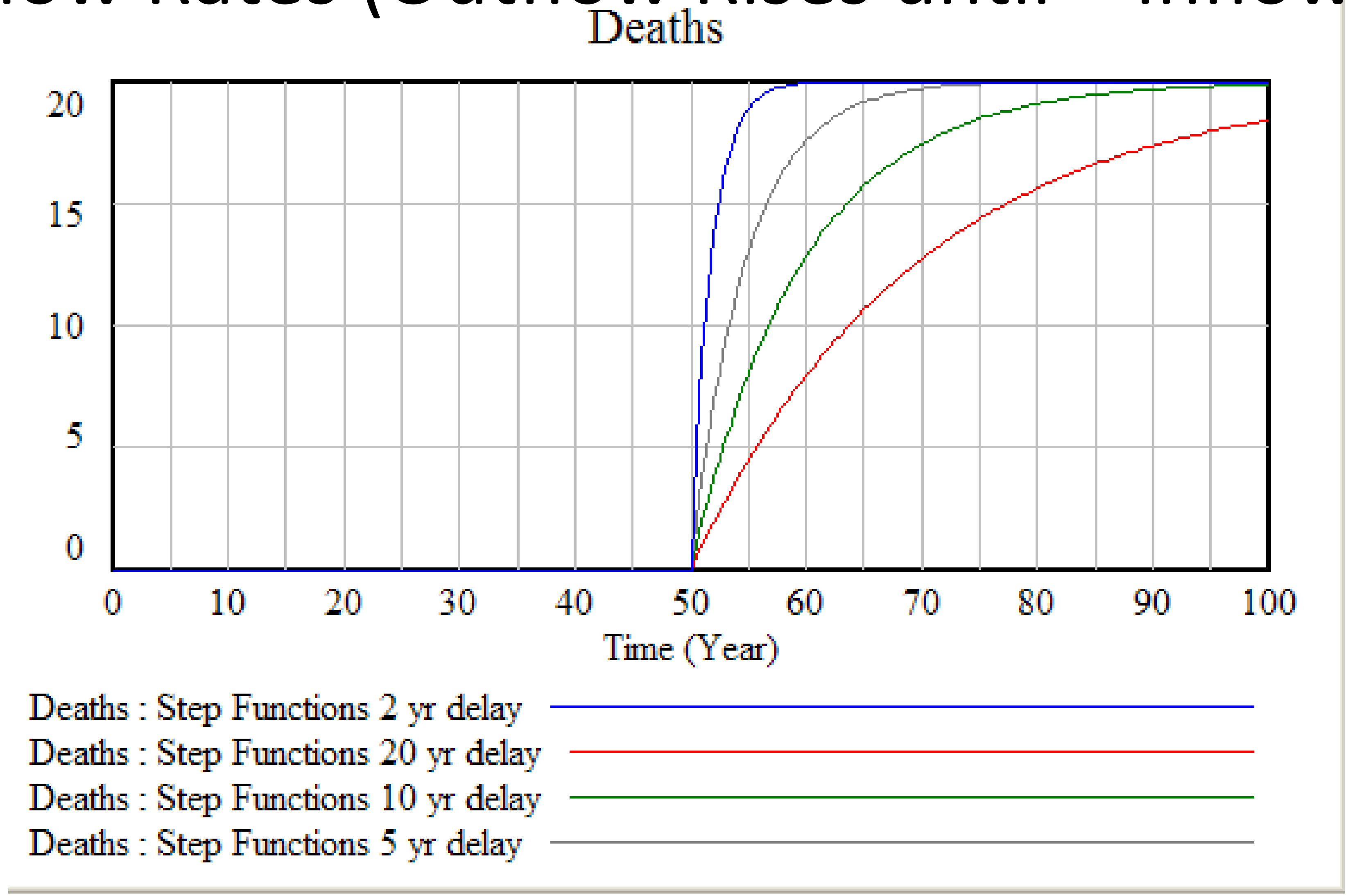
How would this change with alpha?

Stock Starting Empty?
Value of *Stock* (Alpha=.05)



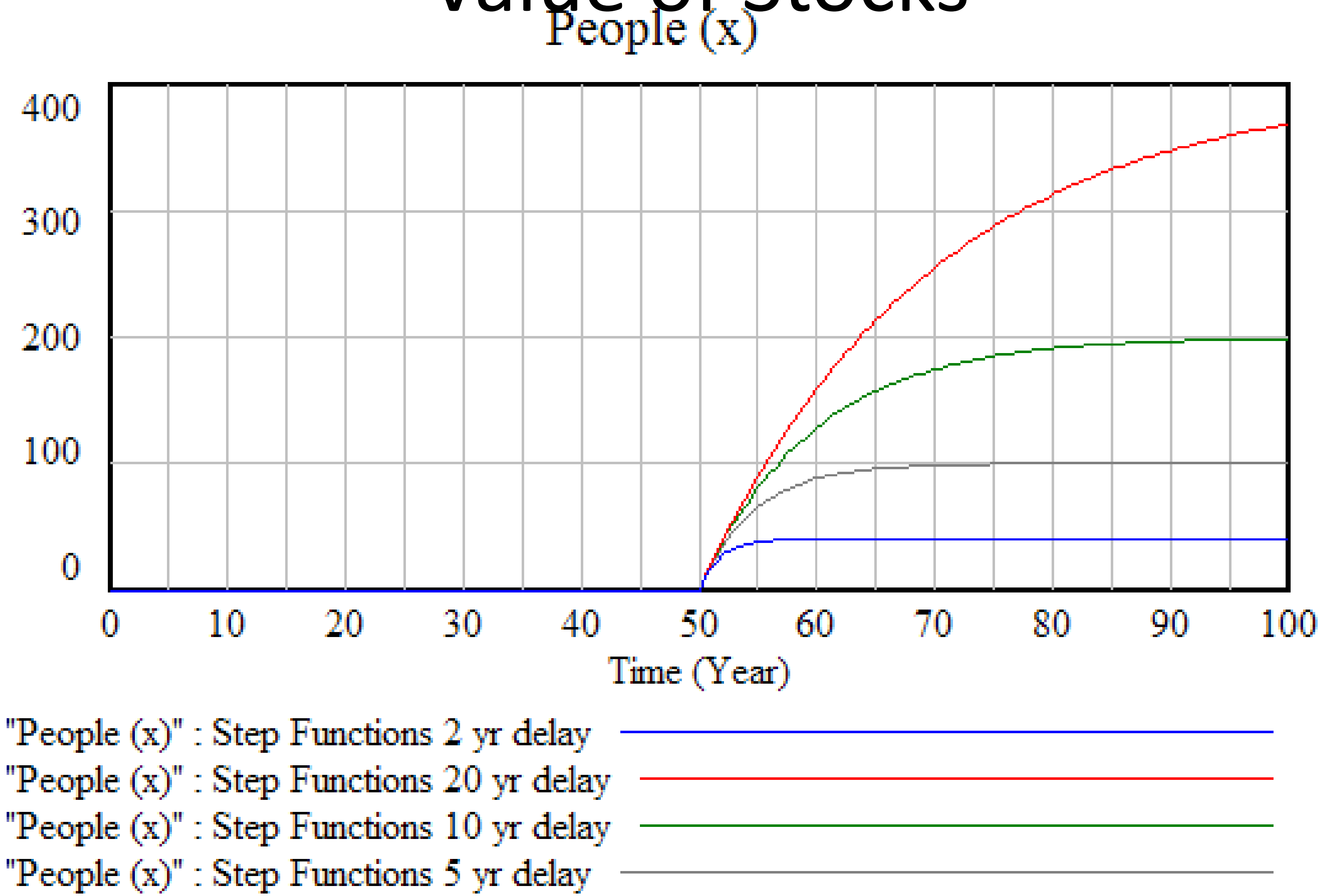
How would this change with alpha?

For Different Values of (1/) Alpha Flow Rates (Outflow Rises until = Inflow)



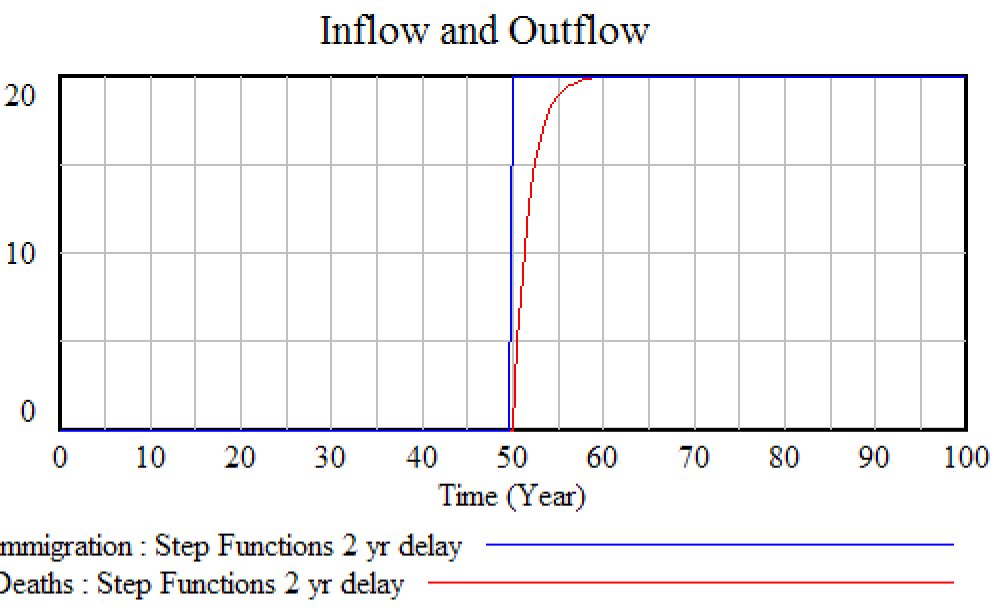
This is for the *flows*. What do stocks do?

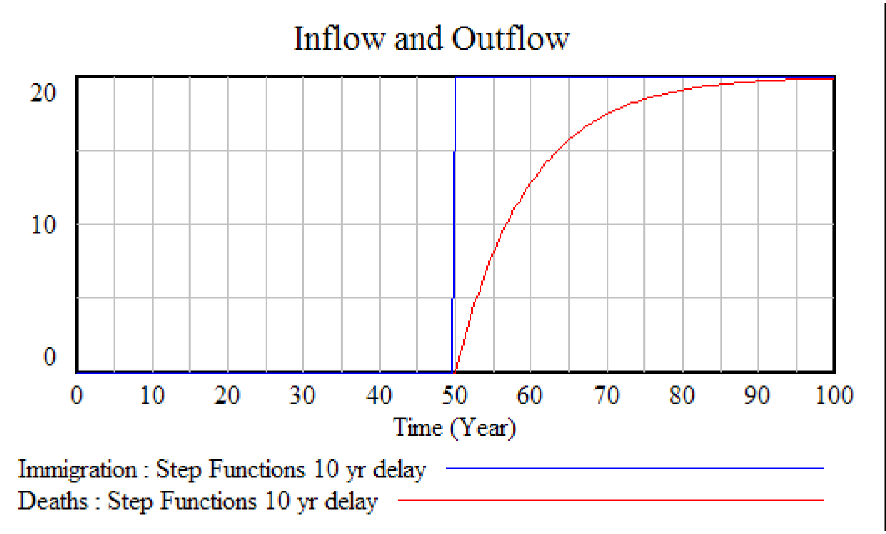
For Different Values of (1/) Alpha
Value of Stocks

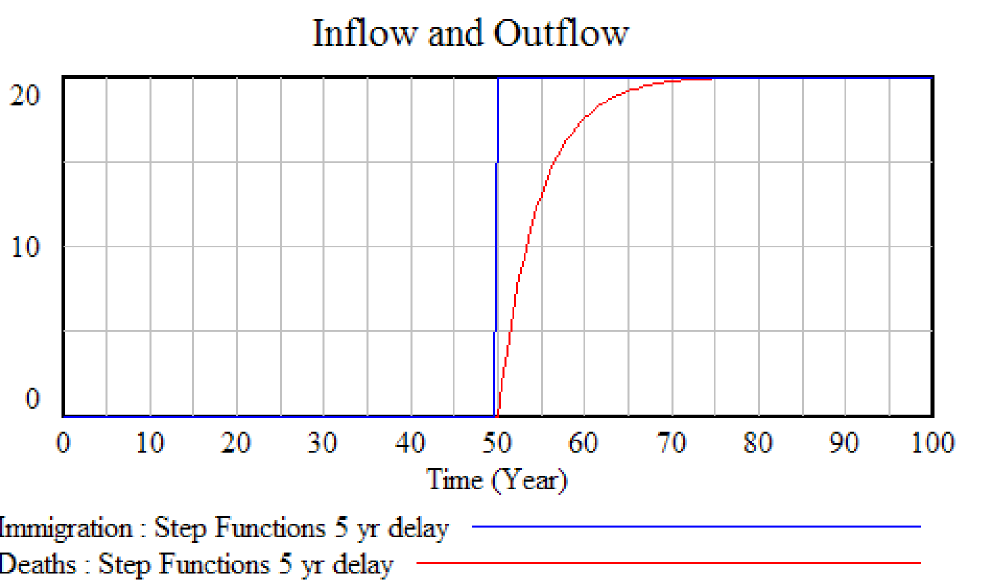


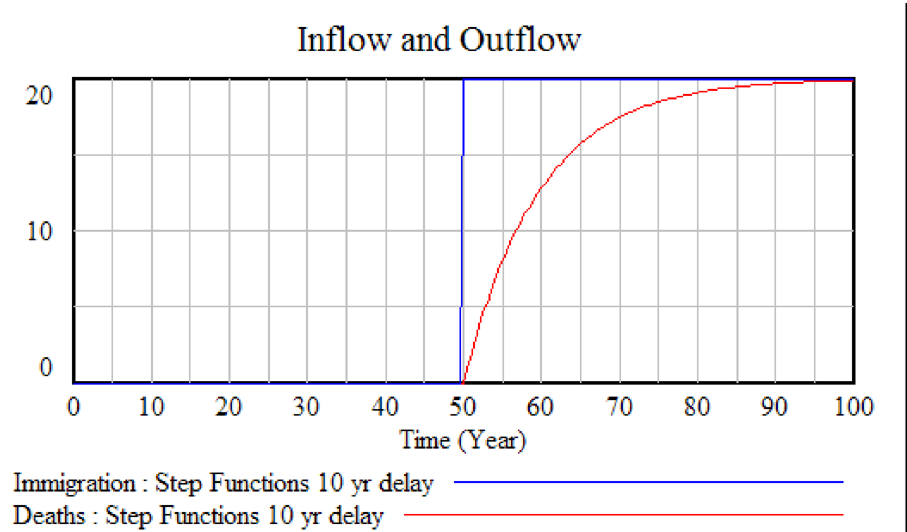
Why do we see this behaviour? A longer time delay (or smaller chance of leaving per unit time) requires *x* to be *larger* to make outflow=inflow

Outflows as Delayed Version of Inputs









Higher Order Delays & Aging Chains

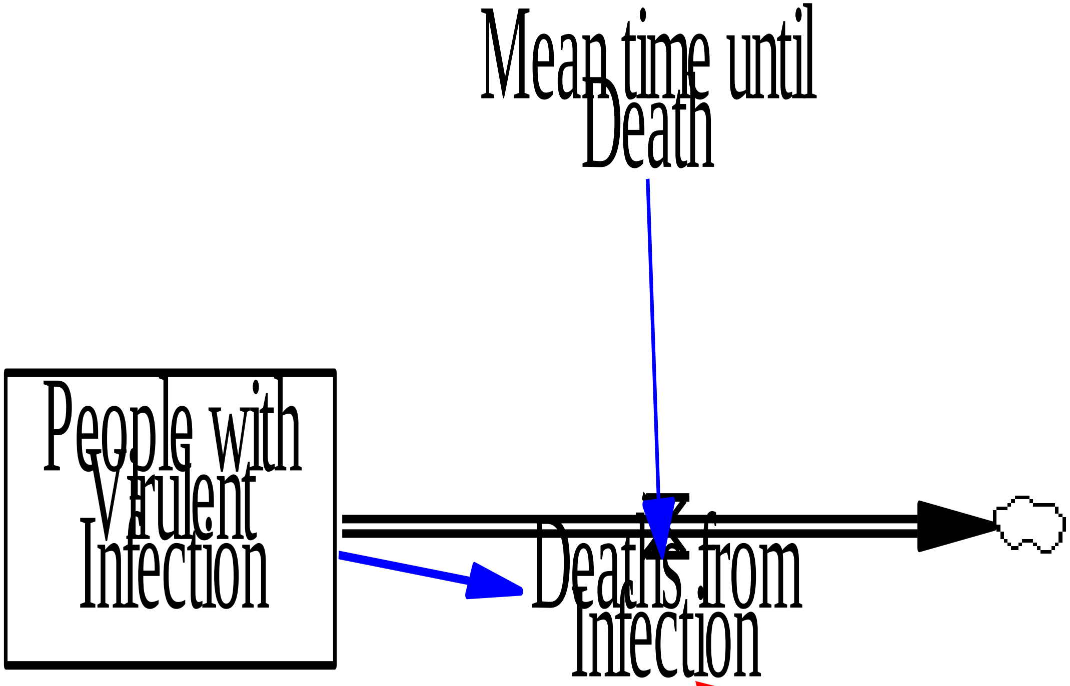
Moving Beyond the “memoryless assumption”

- Recall that first order delays assume that the per-time-unit risk of transitions to the outflow remains equal throughout simulation (i.e. are memoryless)
- Problem: Often we know that transitions are *not* "memoryless” e.g.
  - It may be the transition reflects some physical delays not endogeneously represented (e.g. Slow-growth of bacterial)
  - Buildup of “damage” of high blood sugars (Glycosylation)

Higher Orders of Delays

- We can capture different levels of delay (with increasing levels of fidelity) using cascaded series of 1st order delays
- We call the delay resulting from such a series of *k* 1st order delays a "kth order delay"
  - E.g. 2 first order delays in series yield a 2nd order delay
- The behaviour of a kth order delay is a reflection of the behaviour of the 1st order delays out of which it is built
- To understand the behaviour of kth order delays, we will keep constant the mean time taken to transition across the entire set of all delays

Recall: Simple 1st Order Decay



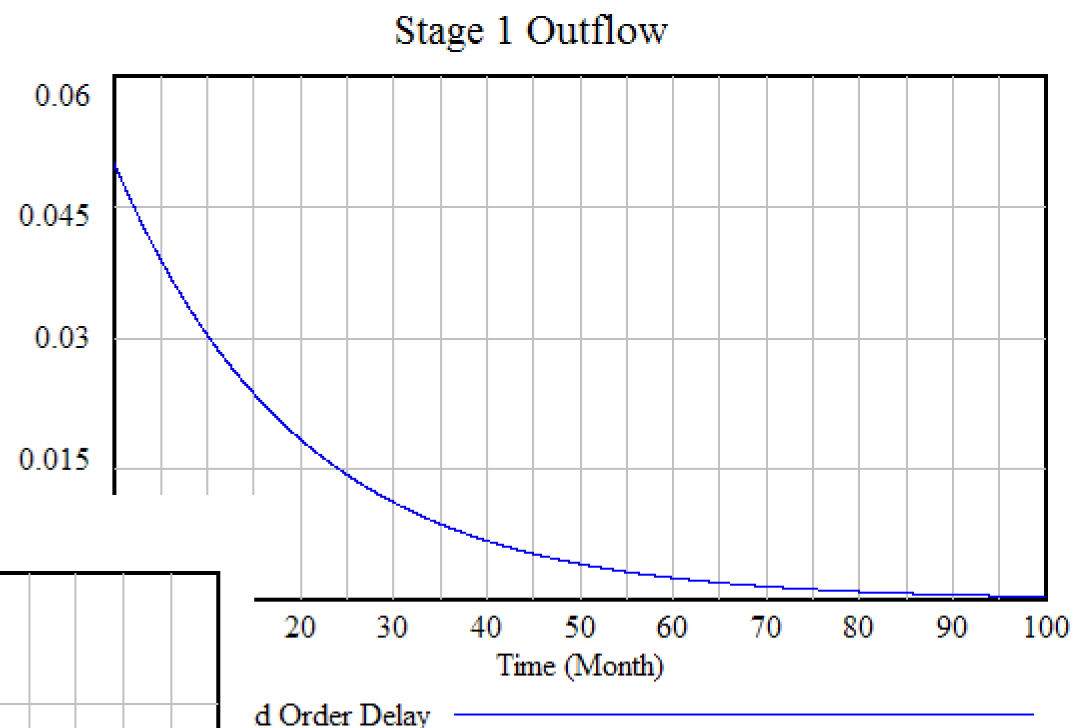
(Initial Value: 1)

Use Formula: People with Virulent Infection/Mean time until Death

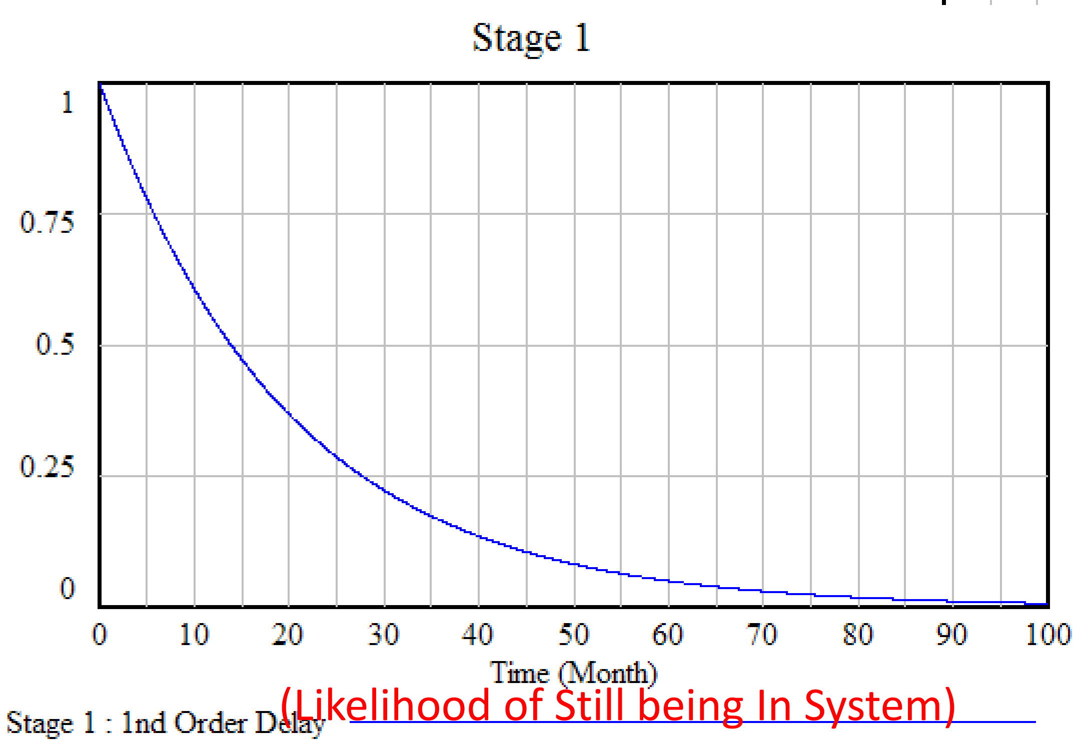
Recall: 1st Order Delay Behaviour

- ***Conditional*** transition prob: For a 1st Order delay, the per-time-unit likelihood of leaving *given that one has not yet left the stock* remains constant
- ***Unconditional*** transition prob: For a 1st Order delay, the unconditional per-time-unit likelihood of leaving declines exponentially
  - i.e. if were were originally in the stock, our chance of having left in the course of a given time unit (e.g. month) declines exponentially
    - This reflects the fact that there are fewer people who could still leave during this time unit!

Recall: 1st Order Delay Behaviour



(Per-month chance of transitioning out during this month)

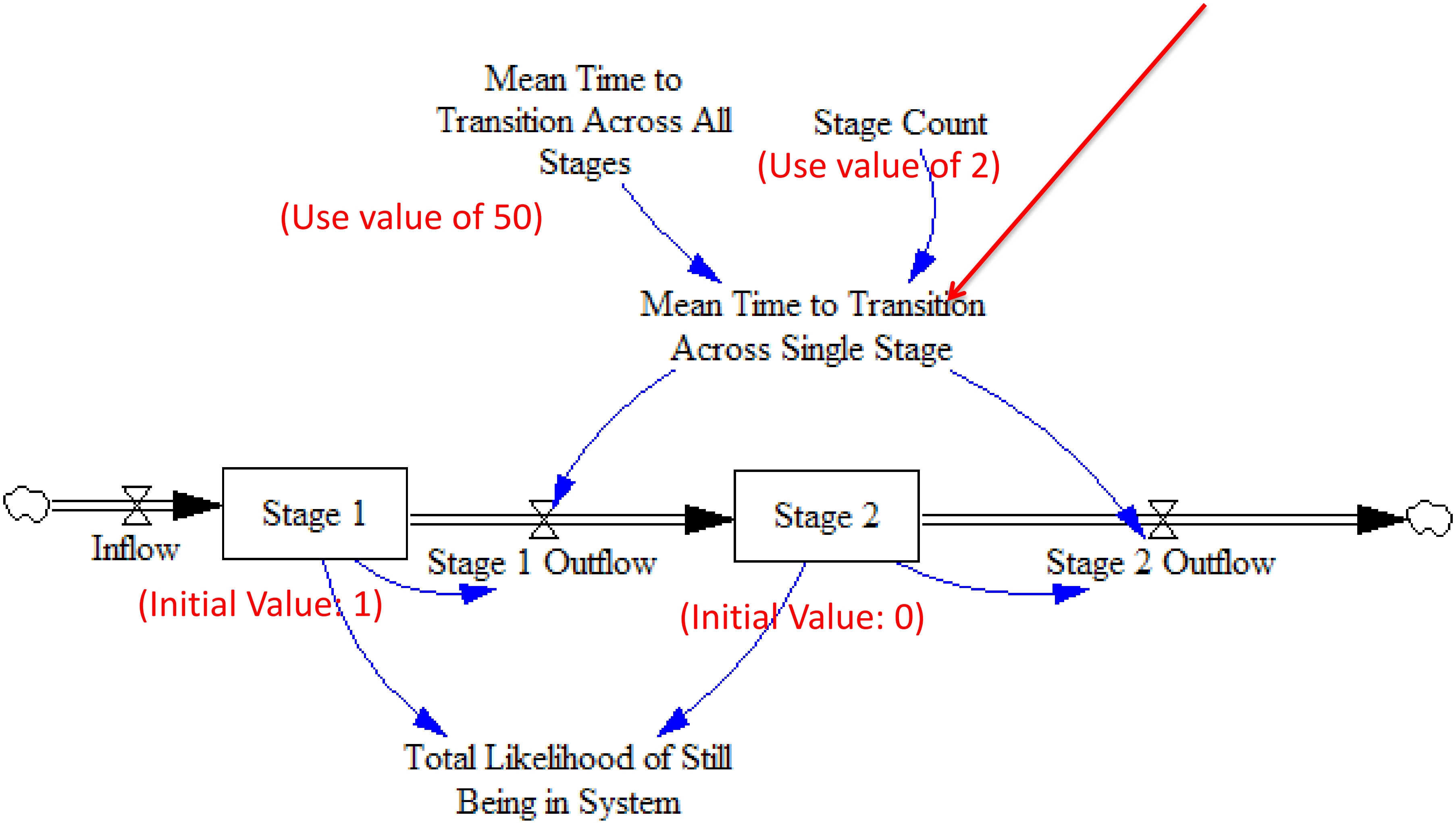


(Likelihood of Still being In System)

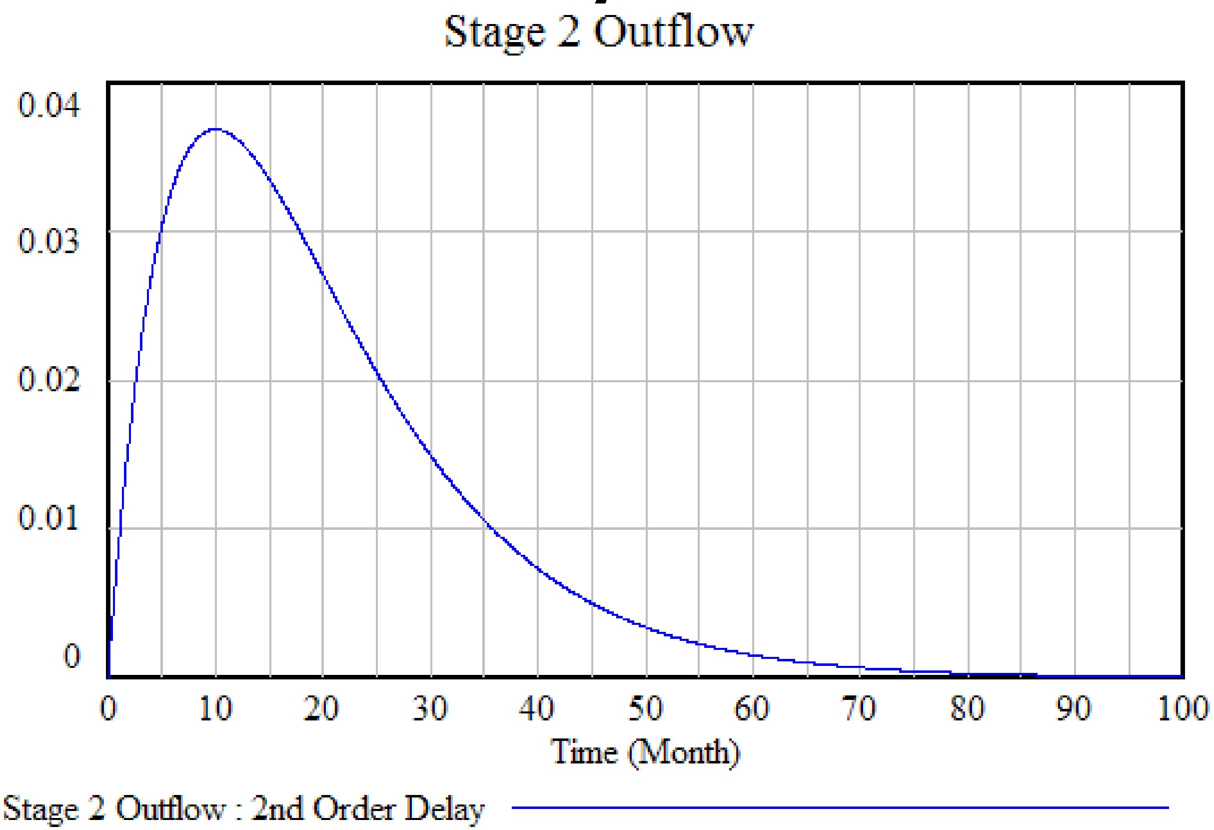
2nd Order Delay

Use Formula:

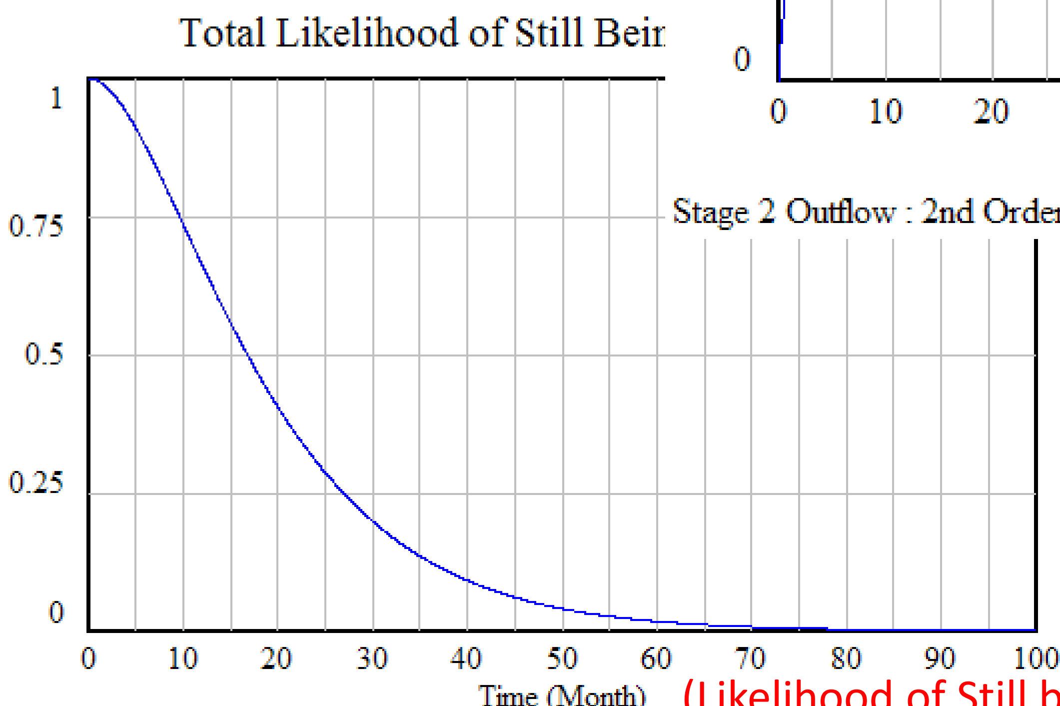
Mean Time to Transition Across All Stages/Stage Count



2nd Order Delay

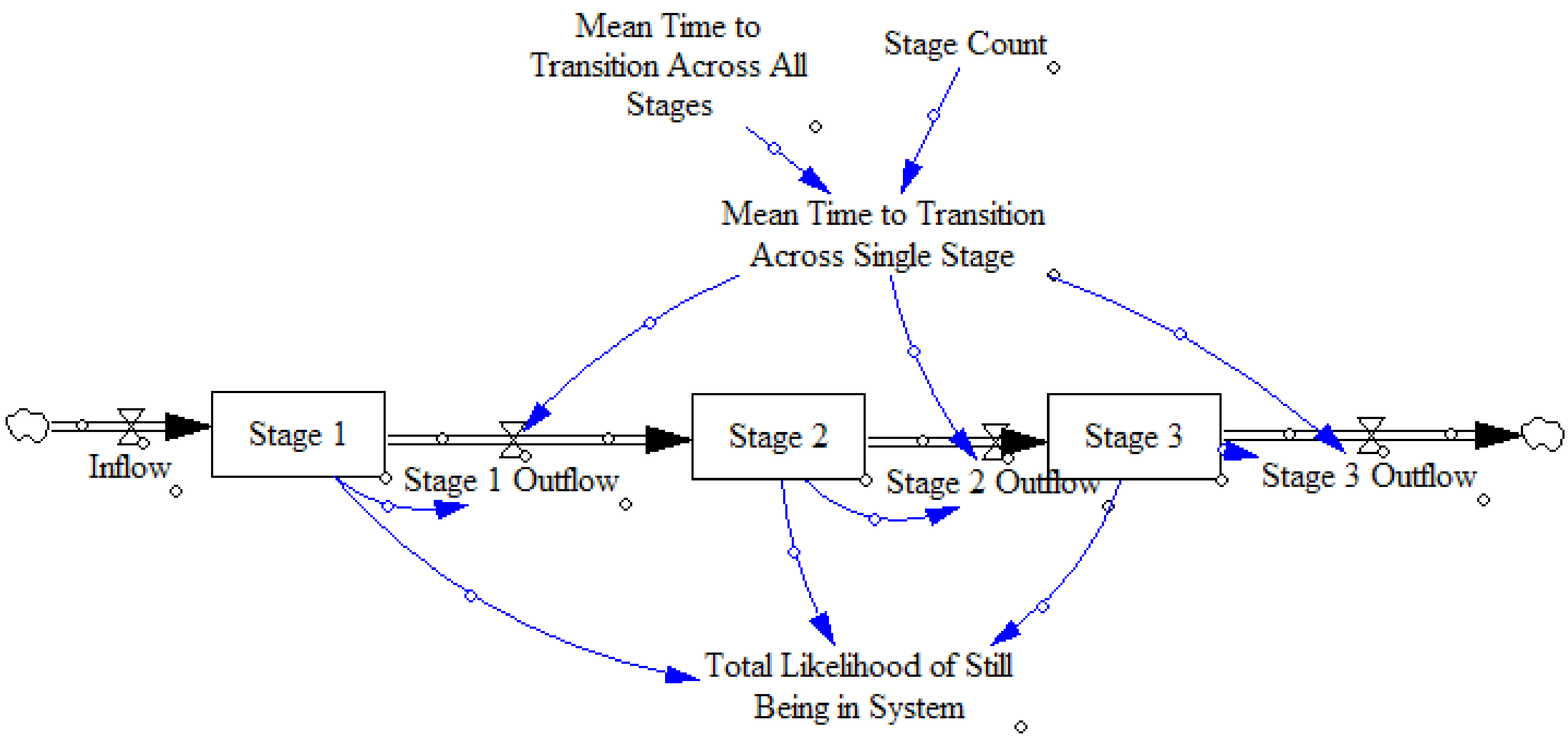


(Per-month chance of transitioning out during this month)

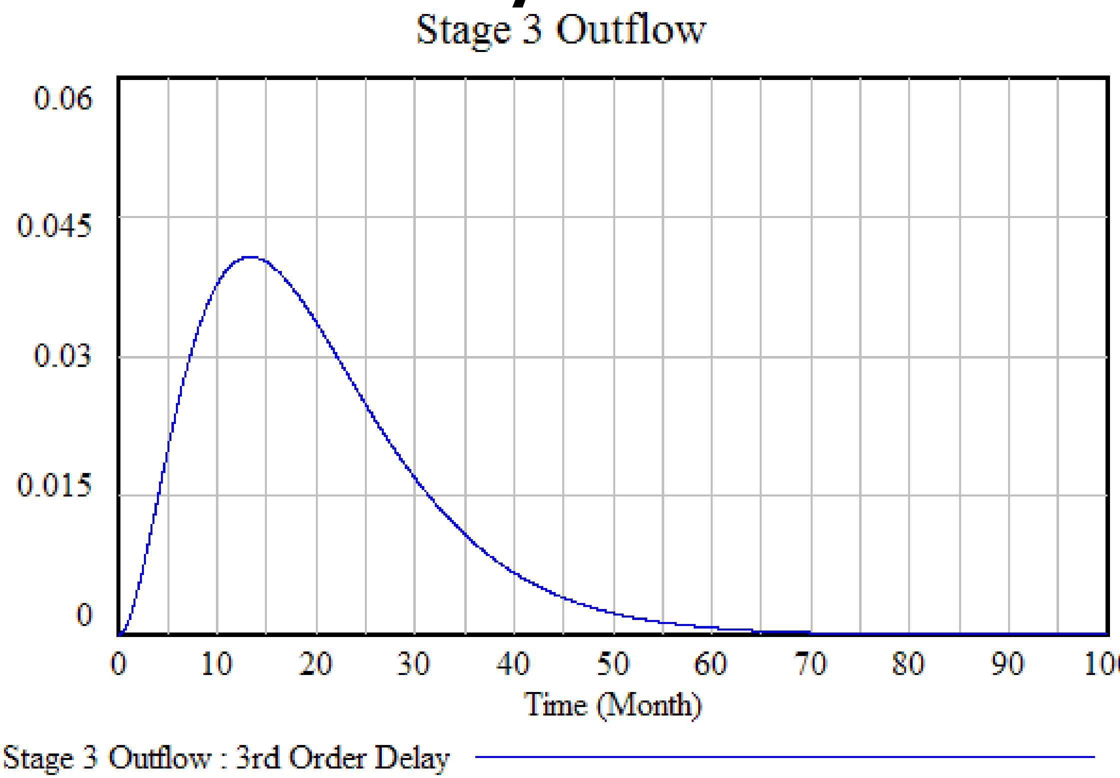


(Likelihood of Still being In System)

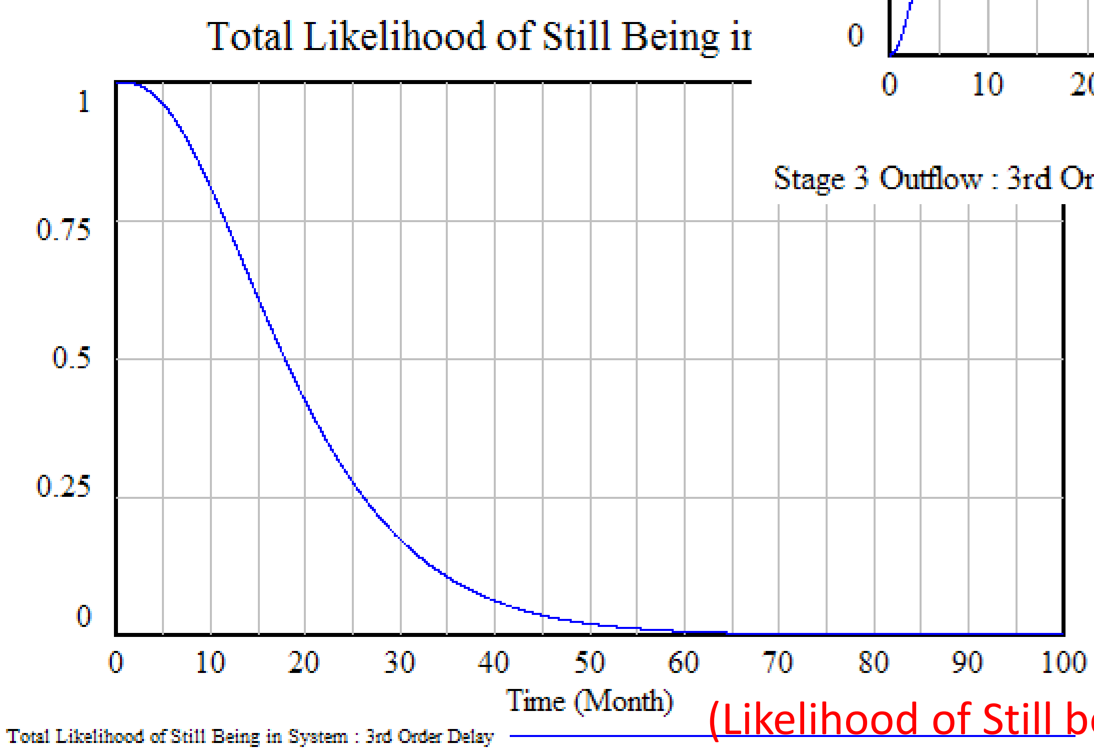
3rd Order Delay



3rd Order Delay

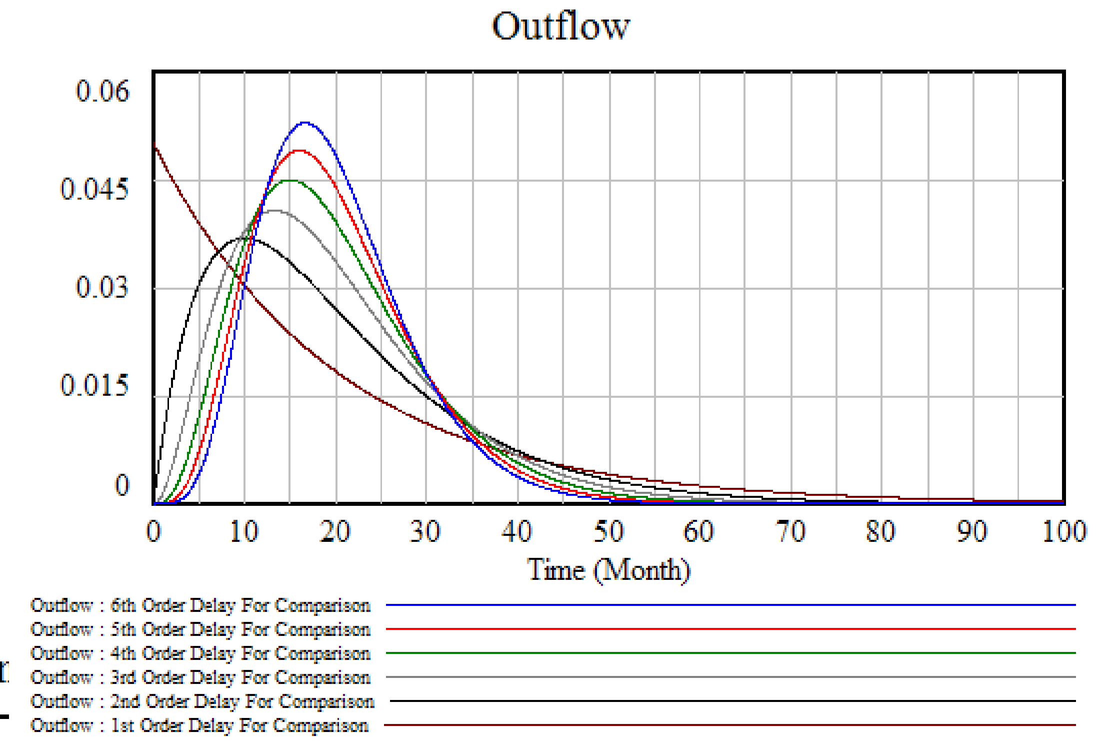


(Per-month chance of transitioning out during this month)

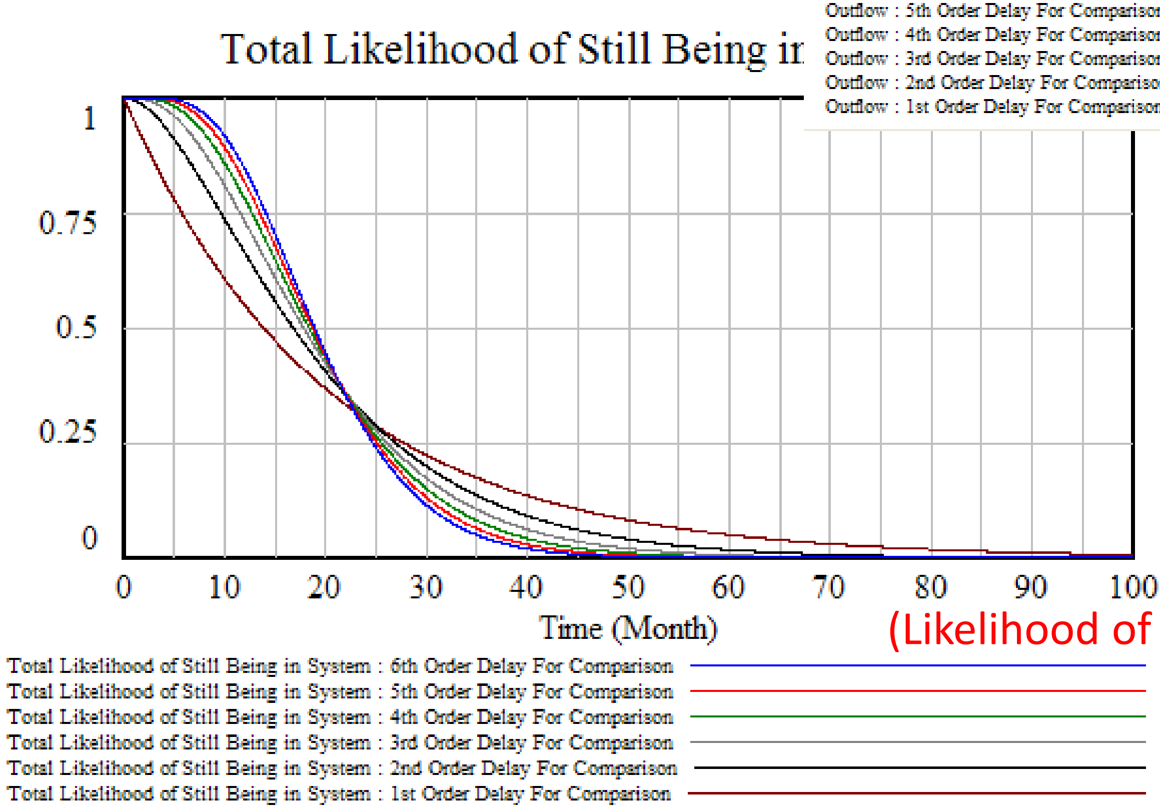


(Likelihood of Still being In System)

1^{st} through 6^{th} OrderDelays



(Per-month chance of transitioning out during this month)



(Likelihood of Still being In System)

Mean Times to Depart Final Stage

- Mean time of k stages is just k times mean time of one stage (e.g. if the mean time for leaving 1 stage requires time μ, mean time for k = $k*\mu$
- In our examples, as we added stages, we reduced the mean time per stage so as to keep the total constant!
  - i.e. if we have k stages, the mean time to leave each stage is 1/k times what it would be with just 1 stage
- Infinite order delay: As we add more and more stages ($k\rightarrow\infty$), the distribution of time to leave the last stage approaches a normal distribution
  - If we reduce the mean time per stage so as to keep the total time constant, this will approach an impulse function
    - This indicates an exactly fixed time to transition through all stages!

Distribution of Time to Depart Final Stage

- The distributions for the total time taken to transition out of the last of k stages are members of the *Erlang* distribution family
  - These are the same as the distribution for the k^{th} interarrival time of a Poisson process
- k=1 gives exponential distribution (first order delay)
- As $k \rightarrow \infty$, approaches normal distribution (Gaussian pdf)

Parameters	$k > 0 \quad \in \mathbb{Z}$ shape $\lambda > 0$ rate (real) alt.: $\theta = 1/\lambda > 0$ scale (real)
Support	$x \in [0; \infty)$
Probability density function (pdf)	$\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$
Cumulative distribution function (cdf)	$\frac{\gamma(k, \lambda x)}{(k-1)!} = 1 - \sum_{n=0}^{k-1} e^{-\lambda x} (\lambda x)^n / n!$
Mean	k/λ
Median	no simple closed form
Mode	$(k-1)/\lambda$ for $k \geq 1$
Variance	k/λ^2
Skewness	$\frac{2}{\sqrt{k}}$
Excess kurtosis	$\frac{6}{k}$
Entropy	$(1-k)\psi(k) + \ln \frac{\Gamma(k)}{\lambda} + k$
Moment-generating function (mgf)	$(1 - t/\lambda)^{-k}$ for $t < \lambda$
Characteristic function	$(1 - it/\lambda)^{-k}$

From Wikipedia, 2009

"Aging Chains" (including successive 1st Order Delays & Competing Risks) in our Model of Chronic Kidney Disease